Dear students, colleagues, and miscellaneous math enthusiasts,

On this page are my class lecture notes for MthSc 208, Ordinary Differential Equations, a class I taught during the Fall 2009 semester at Clemson University. It takes a little more time to write up the my lecture notes neatly, but I feel that it is worth the extra effort, since it benefits everyone involved – me, currente and former students, as well as other students and instructors of differential equation classes. I have put in extra minimal effort to help people who want to use these lecture notes as a guide, such as keeping track of how many lectures I gave each week (which changes due to midterms, holidays, etc). This is the third time I taught this class, each time improving my notes and the content. I am very pleased with how they turned out this time. Feel free to use them if you are teaching your own ODEs class. You aren't required to reference me or my website, but I would appreciate it if you do, especially if you post the pdf files on your webpage.

Here are some of my thoughts on the content of these notes. An ideal differential equations class would be about 25 weeks (instead of the usual 15), and would have linear algebra as a prerequisite. There is simply too much relevant material to fit into a semester, and thus the instructor is forced to pick and choose what topics to cover, no matter what textbook they use. These notes are a combination of the following three excellent texts, as well as things that I have added on my own:

Differential Equations: An Introduction to Modern Methods and Applications by James R. Brannan and William E. Boyce.

Differential Equations by John Polking, Albert Boggess, and David Arnold.

## Introduction to Partial Differential Equations by John Douglas Moore Freely available online: http://www.freescience.info/go.php?pagename=books&id=1706

Please note that while the first two books do not cover PDEs & Fourier series, they have slighly more expensive "with boundary value problems" versions that do (and I recommend using this "upgrade" for anyone teaching ODEs). For each of the 16 weeks of notes, you'll notice that I have marked the corresponding sections in both in the Brannan/Boyce text (BB) and the Polking text (PBA). My course notes can be roughly broken down into the following sections:

- (1) Intro to ODEs (modeling, plotting solutions, and numerical / approximation methods).
- (2) 1st order ODEs
- (3) 2nd order ODEs
- (4)  $2 \times 2$  systems of 1st order ODEs
- (5) Laplace transforms
- (6) Fourier series
- (7) PDEs

This semester differed from my previous ODEs courses in that I did *not* lecture about power series, the method of Frobenius, and abstract vector spaces (I covered  $2 \times 2$  systems instead). There are some other common topics that I have always ommited due to time constraints. These include higher-order systems of ODEs (a lot more work than the  $2 \times 2$  case, and not enough reward for the required extra time, especially since most students haven't had linear algebra), as well as exact differential equations, and theorems about existence and uniqueness of solutions in some epsilon/delta rectangle (all of which I feel is not pertinent enough for engineers). That said, I do emphasize existence and uniqueness of solutions qualitatively, e.g., an  $n^{\text{th}}$  order linear ODE has n "distinct" solutions, and I draw parallels of this to parametrized lines, planes, etc.

Not all ODE classes cover Fourier series and PDEs. Since most of the student are engineers, I feel that this is one of the most important topics, and I teach it in a way that is much different than most engineers do. The material is by nature quite difficult (though very intuitive and conceptual if taught the right way, which it often isn't), and they will surely see it again, so having a few

weeks devoted to it will really help them in their later courses. Additionally (though I am quite biased), I think I present it in the clearest and simplest way possible.

Please note that it is highly unlikely that 149 pages of hand-written notes will contain no errors. As said nicely by Federico Ardila of San Francisco State (who also publicly posts lecture notes): "Everyone makes mistakes when they teach, but not everyone immortalizes them on the internet." That said, if you find an error, or have any other feedback to share, please don't hesitate to email me. Also, I am happy to email you a tarball of the LaTeX files from my homework & exams.

I also have course lecture notes posted from the same ODEs class (MthSc 208), but during the previous semester. Every semester I improve the course organization and content, so if you are using these notes as a reference, I would strongly recommend using the most recent semester. Additionally, I have seen a few minor obvious typos in the earlier versions that I have since forgotten about and probably won't fix. The only exception is if you want to see the difference in content, such as the power series method.

I hope you find these notes useful, students and instructors alike. Feedback is always encouraged.

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