

MthSc 208, Fall 2010 (Differential Equations)

Dr. Matthew Macauley

HW 2

Due Friday August 27th, 2010

- (1) Consider the initial value problem  $y' = t + y$ ,  $y(0) = 1$ .
- (a) When computing a solution by hand using Euler's method, it is beneficial to arrange your work in a table, as shown below where the first step is computed. Continue with Euler's method using step-size  $h = 0.1$  and complete all missing entries of the table.

$k$	$t_k$	$y_k$	$f(t_k, y_k) = t_k + y_k$	$h$	$f(t_k, y_k) \cdot h$
0	0.0	1.0	1.0	0.1	0.1
1	0.1	1.1			
2	0.2				
3	0.3				
4	0.4				
5	0.5				

- (b) Compute the actual value of  $y(0.5)$  by solving the initial value problem  $y' = t + y$ ,  $y(0) = 1$ , and plugging in  $t = 0.5$ .
- (2) Consider the initial value problem  $y' = (1 + t)y$ ,  $y(0) = -1$ .
- (a) Use Euler's method to approximate  $y(1)$ , for step-size  $h = 0.2$ , and then for  $h = 0.1$ . Arrange your results in the tabular form as in the previous exercise.
- (b) Compute the actual value of  $y(1)$  by solving the initial value problem  $y' = (1 + t)y$ ,  $y(0) = -1$  and plugging in  $t = 1$ .
- (3) Solve for  $t$ , and simplify whenever possible.
- (a)  $3e^{-4t} = 5$
- (b)  $2 = e^3 \cdot e^{2t}$
- (c)  $t^2 = e^6$
- (d)  $(\frac{4}{3})^{-t} = 7$
- (e)  $e^{\frac{1}{3} \ln t} = 27$
- (f)  $e^{-\frac{1}{3} \ln t} = 27$
- (4) Compute the following integrals:
- (a)  $\int \frac{1}{2t} dt$
- (b)  $\int \frac{1}{3-4t} dt$
- (5) Find the general solution of the following differential equations.
- (a)  $y' = ty$
- (b)  $ty' = 2y$
- (c)  $y' = e^{t-y}$
- (6) Suppose that \$1200 is invested at a rate of 5%, compounded continuously.
- (a) Assuming no additional withdrawals or deposits, how much will be in the account after 10 years?
- (b) How long will it take the balance to reach \$5000?
- (7) Tritium is an isotope of hydrogen that is sometimes used as a biochemical tracer. Suppose that 100 mg of tritium decays to 80 mg in 4 hours. Determine its half-life.