

MthSc 208, Fall 2010 (Differential Equations)

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HW 4

Due Friday September 3rd, 2010

- (1) Use the integrating factor method to find the general solution of the following differential equations.

(a)  $2y' - 3y = 5$

(b)  $y' + 2ty = 5t$

(c)  $ty' = 4y + t^4$

- (2) Use the variation of parameters method to find the general solution of the following differential equation. Then find the particular solution satisfying the given initial condition.

(a)  $y' - 3y = 4, \quad y(0) = 2$

(b)  $y' + y = e^t, \quad y(0) = 1$

(c)  $y' + 2ty = 2t^3, \quad y(0) = -1.$

- (3) A murder victim is discovered at midnight at the temperature of the body is recorded at  $31^\circ\text{C}$ , and it was discovered that the proportionality constant in Newton's law was  $k = \ln(5/4) \approx 0.223$ . Assume that at midnight the surrounding air temperature  $A(t)$  is  $0^\circ\text{C}$ , and is falling at a constant rate of  $1^\circ\text{C}$  per hour. At what time did the victim die? (Set  $T(t) = 37$  and solve for  $t$  – use a computer or calculator for this part.) *Hint:* Letting  $t = 0$  represent midnight will simplify your calculations.

- (4) Consider Newton's law of cooling, but suppose that the ambient temperature varies sinusoidally with time, as in

$$T' = k(A \sin \omega t - T).$$

- (a) Solve the homogeneous equation,  $T'_h = -kT_h$ .

- (b) The ODE above is not autonomous, so finding a particular solution  $T_p$  is a bit more difficult. However, it doesn't hurt to guess. As a first guess, substitute  $T_p = C \cos \omega t + D \sin \omega t$  into the equation  $T' + kT = kA \sin \omega t$  and show that

$$-\omega C + kD = kA \quad \text{and} \quad kC + \omega D = 0.$$

- (c) Give a qualitative description of what this particular solution represents [Hint: What would happen if initially, the object is the same as the ambient temperature?]

- (d) Solve the simultaneous equations in part (b), and determine the general solution to this ODE.

- (5) Suppose that the temperature  $T$  inside a mountain cabin behaves according to Newton's law of cooling, as in

$$\frac{dT}{dt} = \frac{1}{2}(A - T),$$

where  $t$  is measured in hours and the ambient temperature  $A(t)$  outside the cabin varies sinusoidally with a period of 24 hours. At 6am, the ambient temperature outside is at a minimum of  $40^\circ$ , and at 6pm, the ambient temperature is at a maximum of  $80^\circ$ .

- (a) Adjust the differential equation above to model the sinusoidal nature of the ambient temperature.
- (b) Suppose that at midnight the temperature inside the cabin is  $50^\circ$ . Solve the resulting initial value problem. [Hint: Letting  $t = 0$  represent midnight will simplify your calculations.]
- (c) Sketch the graph of the temperature inside the cabin. On the same coordinate system, superimpose the plot of the ambient temperature outside the cabin. Comment on the appearance of the plot.
- (6) Solve the differential equation  $y' = 2y + 4$  four different ways:
- (a) Finding a constant solution, and writing  $y(t) = y_h(t) + y_p(t)$ .
- (b) Integrating factor
- (c) Variation of parameters
- (d) Separation of variables