MthSc 208, Fall 2010 (Differential Equations)
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HW 7
Due Tuesday September 14th, 2010

(1) Find the particular solution to the following 2^{nd} order linear homogeneous ODEs.
  (a) $y'' - y' - 2y = 0, \quad y(0) = -1, \quad y'(0) = 2$
  (b) $y'' - 4y' - 5y = 0, \quad y(1) = -1, \quad y'(1) = -1$
  (c) $y'' + 25y = 3, \quad y(0) = 1, \quad y'(0) = -1$
  (d) $y'' - 2y' + 17y = 0, \quad y(0) = -2, \quad y'(0) = 3$

(2) Find the general solution to the following 2^{nd} order linear inhomogeneous ODEs, by solving the associated homogeneous equation, and then finding a constant (particular) solution.
  (a) $y'' + y' - 12y = 24$
  (a) $y'' = -4y + 3$

(3) As we've seen, to solve ODE of the form

$$y'' + py' + qy = 0\,, \qquad p \text{ and } q \text{ constants}$$

we assume that the solution has the form e^{rt}, and then we plug this back into the ODE to get the *characteristic equation*: $r^2 + pr + q = 0$. Given that this equation has a double root $r = r_1$ (i.e., the roots are $r_1 = r_2$), show by direct substitution that $y = te^{rt}$ is a solution of the ODE, and then write down the general solution. [Hint: If there's a double-root, then it must be $-\frac{p}{2}$. Why?]

(4) Suppose that $z(t) = x(t) + iy(t)$ is a solution of

$$z'' + pz' + qz = Ae^{i\omega t}\,.$$

Substitute $z(t)$ into this equation above. Then compare (equate) the real and imaginary parts of each side to prove two facts:

$$x'' + px' + qx = A\cos\omega t$$
$$y'' + py' + qy = A\sin\omega t\,.$$

Write a sentence or two summarizing the significance of this result.

(5) Solve the following initial value problems using the method of undetermined coefficients.
  (a) $y'' + 3y' + 2y = -3e^{-4t}, \quad y(0) = 1, \quad y'(0) = 0$
  (b) $y'' + 2y' + 2y = 2\cos 2t, \quad y(0) = -2, \quad y'(0) = 0$
  (c) $y'' + 4y' + 4y = 4 - t, \quad y(0) = -1, \quad y'(0) = 0$
  (d) $y'' - 2y' + y = t^3, \quad y(0) = 1, \quad y'(0) = 0$