MthSc 208, Fall 2010 (Differential Equations)
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HW 8
Due Tuesday September 21st, 2010

(1) If $y_f(t)$ is a solution of
$$y'' + py' + qy = f(t)$$
and $y_g(t)$ is a solution of
$$y'' + py' + qy = g(t)\,,$$
show that $z(t) = \alpha y_f(t) + \beta y_g(t)$ is a solution of
$$y'' + py' + qy = \alpha f(t) + \beta g(t)\,,$$
where α and β are any real numbers, by plugging it into the ODE.

(2) Find the general solution to the following 2^{nd} order linear inhomogeneous ODEs.
  (a) $y'' + 2y' + 2y = 2 + \cos 2t$
  (b) $y'' + 25y = 2 + 3t + 4\cos 2t$
  (b) $y'' - y = t - e^{-t}$.

(3) (a) Find the general solution of $y'' + 3y' + 2y = te^{-4t}$. (Look for a particular solution of the form $y_p = (at + b)e^{-4t}$.)
  (b) Use a similar approach as above to find a solution to the differential equation $y'' + 2y' + y = t^2 e^{-2t}$.

(4) Find the general solution of $y'' + 2y' + 2y = e^{-2t}\sin t$. (Look for a particular solution of the form $y_p = e^{-2t}(a\cos t + b\sin t)$.)

(5) For the following exercises, rewrite the given function in the form
$$y = A\cos(\omega t - \phi) = A\cos\left(\omega\left(t - \frac{\phi}{\omega}\right)\right)\,,$$
and then plot the graph of this function.
  (a) $y = \cos 2t + \sin 2t$
  (b) $y = \cos t - \sin t$
  (c) $y = \cos 4t + \sqrt{3}\sin 4t$
  (d) $y = -\sqrt{3}\cos 2t + \sin 2t$.

(6) Consider the undamped oscillator
$$mx'' + kx = 0, \qquad x(0) = x_0, \qquad x'(0) = v_0.$$
  (a) Write the particular solution of this initial value problem in the form $x(t) = a\cos\omega t + b\sin\omega t$ (i.e., determine a, b, and ω.).
  (b) Write you solution in the form $x(t) = A\cos(\omega t - \phi)$ (i.e., determine A).