MthSc 208, Fall 2010 (Differential Equations)
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HW 12
Due Tuesday October 5th, 2010

(1) Find the general solution for each of the given system of equations. Draw a phase portrait. Describe the behavior of the solutions as $t \to \infty$.

(a) $\mathbf{x}' = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix} \mathbf{x}$ (b) $\mathbf{x}' = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} \mathbf{x}$

(c) $\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} \mathbf{x}$ (d) $\mathbf{x}' = \begin{pmatrix} 4 & -3 \\ 8 & -6 \end{pmatrix} \mathbf{x}$

(2) In each of the next four problems, the eigenvalues and eigenvectors of a matrix A are given. Consider the corresponding system $\mathbf{x}' = \mathbf{A}\mathbf{x}$. Without using a computer, draw each of the following graphs.

(i) Sketch a phase portrait of the system.
(ii) Sketch the trajectory passing through the initial point $(2, 3)$.
(iii) For the trajectory in part (ii), sketch the component plots of x_1 versus t and x_2 versus t on the same set of axes.

(a) $\lambda_1 = -1, \quad \mathbf{v}_1 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}; \qquad \lambda_2 = -4, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$

(b) $\lambda_1 = 1, \quad \mathbf{v}_1 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}; \qquad \lambda_2 = -4, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$

(c) $\lambda_1 = -1, \quad \mathbf{v}_1 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}; \qquad \lambda_2 = 4, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$

(d) $\lambda_1 = 1, \quad \mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}; \qquad \lambda_2 = 4, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}.$

(3) In each of the next four problems, the eigenvalues and eigenvectors of a matrix A are given. Consider the corresponding system $\mathbf{x}' = \mathbf{A}\mathbf{x}$. Without using a computer, draw each of the following graphs.

(i) Sketch a phase portrait of the system.
(ii) Sketch the trajectory passing through the initial point $(2, 3)$.

(a) $\lambda_1 = -4, \quad \mathbf{v}_1 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}; \qquad \lambda_2 = -1, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$

(b) $\lambda_1 = 4, \quad \mathbf{v}_1 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}; \qquad \lambda_2 = -1, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$

(c) $\lambda_1 = -4, \quad \mathbf{v}_1 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}; \qquad \lambda_2 = 1, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$

(d) $\lambda_1 = 4, \quad \mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}; \qquad \lambda_2 = 1, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}.$

(4) Find the general solution for each of the given systems in terms of real-valued function, and draw a phase portrait. Describe the behavior of the solutions as $t \to \infty$.

(a) $\mathbf{x}' = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix} \mathbf{x}$ (b) $\mathbf{x}' = \begin{pmatrix} 1 & -1 \\ 5 & -3 \end{pmatrix} \mathbf{x}$ (c) $\mathbf{x}' = \begin{pmatrix} 1 & 2 \\ -5 & -1 \end{pmatrix} \mathbf{x}$

(5) In the problems below, the coefficient matrix contains a parameter α.

(a) Determine the eigenvalues in terms of α.
(b) Find the critical value or values of α where the qualitative nature of the phase portrait for the system changes.
(c) Draw a phase portrait for a value of α slight below, and for another value slightly above, each critical value.
(d) Draw a phase portrait when α is exactly the critical value.

(a) $\mathbf{x}' = \begin{pmatrix} \alpha & 1 \\ -1 & \alpha \end{pmatrix} \mathbf{x}$ (b) $\mathbf{x}' = \begin{pmatrix} -1 & \alpha \\ -1 & -1 \end{pmatrix} \mathbf{x}$

(6) Find the general solution for each of the given systems and draw a phase portrait. Describe the behavior of the solutions as $t \to \infty$.

(a) $\mathbf{x}' = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} \mathbf{x}$ (b) $\mathbf{x}' = \begin{pmatrix} -3/2 & 1 \\ -1/4 & -1/2 \end{pmatrix} \mathbf{x}$ (c) $\mathbf{x}' = \begin{pmatrix} -1 & -1/2 \\ 2 & -3 \end{pmatrix} \mathbf{x}$