MthSc 208, Fall 2010 (Differential Equations)
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HW 13
Due Friday October 15th, 2010

(1) Find the Laplace transform of the following functions by explicitly computing $\int_0^\infty f(t)\, e^{-st}\, dt$.
   (a) $f(t) = 3$
   (b) $f(t) = e^{3t}$
   (c) $f(t) = \cos 2t$
   (d) $f(t) = te^{2t}$
   (e) $f(t) = e^{-3t} \sin 2t$

(2) Sketch each of the following piecewise defined functions, and compute their Laplace transforms.
   (a) $f(t) = \begin{cases} 0, & 0 \le t < 4 \\ 5, & t \ge 4 \end{cases}$ (b) $f(t) = \begin{cases} t, & 0 \le t < 3 \\ 3, & t \ge 3 \end{cases}$

(3) Engineers frequently use the *Heavyside function*, defined by

$$H(t) = \begin{cases} 0, & t < 0 \\ 1, & t \ge 0 \end{cases}$$

to emulate turning on a switch at a certain instance in time. Sketch the graph of the function $x(t) = e^{0.2t}$ and compute its Laplace transform, $X(s)$. On a different set of axes, sketch the graph of

$$y(t) = H(t-3)e^{0.2t}$$

and calculate its Laplace transform, $Y(s)$. How do $X(s)$ and $Y(s)$ differ? What do you think the Laplace transform of $H(t-c)e^{0.2t}$ is, where c is an arbitrary positive constant?

(4) Find the Laplace transform of the following functions by using a table of Laplace transforms
   (a) $f(t) = -2$
   (b) $f(t) = e^{-2t}$
   (c) $f(t) = \sin 3t$
   (d) $f(t) = te^{-3t}$
   (e) $f(t) = e^{2t} \cos 2t$

(5) Transform the given initial value problem into an algebraic equation involving $Y(s) := \mathcal{L}(y)$, and solve for $Y(s)$.
   (a) $y'' + y = \sin 4t,\ \ y(0) = 0,\ \ y'(0) = 1$
   (b) $y'' + y' + 2y = \cos 2t + \sin 3t,\ \ y(0) = -1,\ \ y'(0) = 1$
   (c) $y' + y = e^{-t} \sin 3t,\ \ y(0) = 0$

(6) Find the inverse Laplace transform of the following functions.
   (a) $Y(s) = \dfrac{2}{3-5s}$
   (b) $Y(s) = \dfrac{1}{s^2+4}$
   (c) $Y(s) = \dfrac{5s}{s^2+9}$
   (d) $Y(s) = \dfrac{3}{s^2}$
   (e) $Y(s) = \dfrac{3s+2}{s^2+25}$
   (f) $Y(s) = \dfrac{2-5s}{s^2+9}$
   (g) $Y(s) = \dfrac{s}{(s+2)^2+4}$
   (h) $Y(s) = \dfrac{3s+2}{s^2+4s+29}$

(i) $Y(s) = \dfrac{2s - 2}{(s-4)(s+2)}$

(j) $Y(s) = \dfrac{3s^2 + s + 1}{(s-2)(s^2+1)}$