MthSc 208, Fall 2010 (Differential Equations)
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HW 18
Due Friday November 12th, 2010

(1) (a) Find the complex Fourier coefficients of the function

$$f(x) = x^2 \qquad \text{for } -\pi < x \leq \pi,$$

extended to be periodic of period 2π.

(b) Find the real form of the Fourier series. *Hint: Use* $a_n = c_n + c_{-n}$, *and* $b_n = i(c_n - c_{-n})$.

(2) Compute the complex Fourier series for the function defined on the interval $[-\pi, \pi]$:

$$f(x) = \begin{cases} -1, & -\pi \leq x < 0, \\ 4, & 0 \leq x \leq \pi. \end{cases}$$

Use the c_n's to find the coefficients of the real Fourier series (the a_n's and b_n's).

(3) Find the real and complex Fourier series for the function defined on the interval $[-\pi, \pi]$:

$$f(x) = \begin{cases} 0, & -\pi \leq x < 0, \\ 1, & 0 \leq x \leq \pi. \end{cases}$$

Only compute one of these directly (your choice), and then use the formulas relating the real and complex coefficients to compute the other.

(4) Compute the complex Fourier series for the function $f(x) = \pi - x$ defined on the interval $[-\pi, \pi]$. Use the c_n's to to find the coefficients of the real version of the Fourier series.

(5) Prove Parseval's identity:

$$\frac{1}{\pi}\int_{-\pi}^{\pi} (f(x))^2\, dx = \frac{1}{2}a_0^2 + \sum_{n=1}^{\infty}(a_n^2 + b_n^2)\ .$$

(6) Use Parseval's identity, and the Fourier series of the function $f(x) = x^2$ on $[-\pi, \pi]$, to compute $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^4}$.

(7) Compute $\displaystyle\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}$. *Hint*: Compute the Fourier series for $f(x) = |x|$, and then look at $f(\pi)$. (Parseval's identity not needed!)