# Week 4 Summary

- **Logistic equation:** $y' = ry\left(1 - \frac{y}{M}\right)$, $\quad y(t) = \dfrac{M}{1 + Ce^{rt}}$
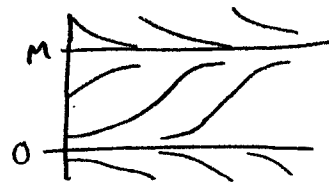
  Threshold equation: $y' = -ry\left(1 - \frac{y}{T}\right)$
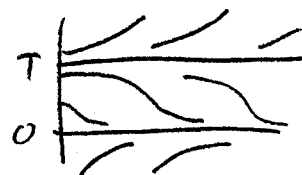
  Put these together:

  $$y' = -ry\left(1 - \frac{y}{M}\right)\left(1 - \frac{y}{T}\right)$$
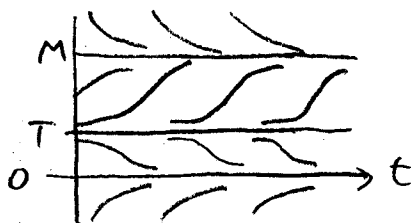
- **2nd order linear ODEs:** $\quad y'' + p(t)\,y' + q(t)\,y = f(t).$

  Homogeneous if $f(t) = 0$.

  General solution: $y(t) = y_h(t) + y_p(t) = C_1 y_1(t) + C_2 y_2(t) + y_p(t)$

- **Constant coefficients:** $\quad y'' + py' + qy = 0$

  Assume $y(t) = e^{rt}$, plug back in and solve for r

  Get $e^{rt}(r^2 + pr + q) = 0$.

  3 cases: (i) $r_1 \neq r_2$ real. $\quad y(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t}$

  (ii) $r_1 = r_2$ $\qquad\qquad y(t) = C_1 e^{r_1 t} + C_2 t e^{r_1 t}$

  (iii) $r_{1,2} = a \pm bi$ $\qquad y(t) = e^{at}\left(A \cos bt + B \sin bt\right).$

- **Inhomogeneous equations:** $\quad y'' + py' + qy = f(t).$

  Guess that $y_p(t)$ "has the same form" as $f(t)$

  This is the "method of undetermined coefficients."