MTHSC 208, Fall 2010

Week 6 summary:

Basic linear algebra:

* A system of 2 linear equations $\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases}$

  can be written as $A\vec{x} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \vec{b}$.

* $A\vec{x} = \vec{b}$ has a unique solution iff $\det A := a_{11}a_{22} - a_{12}a_{21} \neq 0$.

* The <u>inverse</u> of A exists iff $\det A \neq 0$, and is $A^{-1} = \frac{1}{\det A}\begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix}$;
  and $AA^{-1} = A^{-1}A = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, the <u>identity</u> matrix.

* For any matrix A, $AI = IA = A$. Thus, if $\det A \neq 0$,
  we can solve $A\vec{x} = \vec{b}$ by $\vec{x} = A^{-1}A\vec{x} = A^{-1}\vec{b}$.

* If $A\vec{v} = \lambda\vec{v}$, then $\vec{v}$ is an <u>eigenvector</u> for A with eigenvalue λ.

  To find λ, solve $\det(A - \lambda I) = 0$ for λ.

  To find $\vec{v}$, solve $(A - \lambda I)\vec{v} = 0$ for $\vec{v}$.

  <u>Note</u>: $\det(A - \lambda I) = \lambda^2 - (\operatorname{tr} A)\lambda + (\det A) = 0$, $\operatorname{tr} A = a_{11} + a_{22}$.