MTHSC 208, Fall 2010

Week 7 summary

- We can write a system of two 1^{st} order ODE's

$$\begin{cases} x_1' = a_{11}x_1 + a_{12}x_2 + b_1 & x_1(t_0) = d_1 \\ x_2' = a_{21}x_1 + a_{22}x_2 + b_2 & x_2(t_0) = d_2 \end{cases}$$

as

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}, \quad \begin{pmatrix} x_1(t_0) \\ x_2(t_0) \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}, \quad \text{or just}$$

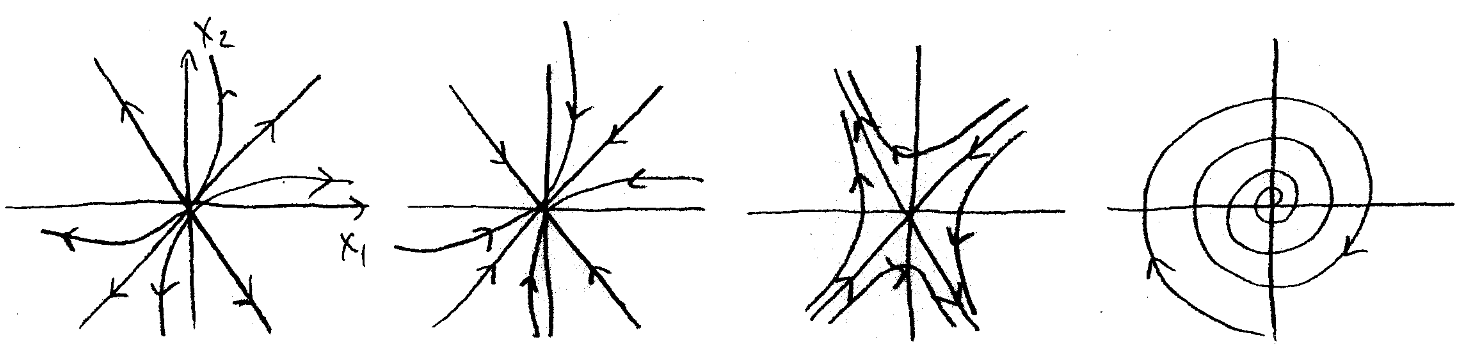
$$\vec{x}' = A\vec{x} + \vec{b}, \quad \vec{x}(t_0) = \vec{d}.$$

- When the eigenvalues of A are distinct, the general solution of $\vec{x}' = A\vec{x}$ is $\vec{x}(t) = C_1 e^{\lambda_1 t}\vec{v}_1 + C_2 e^{\lambda_2 t}\vec{v}_2$.
- If $\lambda_{1,2} = a \pm bi$, then $\vec{x}(t) = C_1\vec{x}_1(t) + C_2\vec{x}_2(t)$,

  where $\vec{x}_1(t) = e^{(a+bi)t}\vec{v} = e^{at}(\cos bt + i\sin bt)\vec{v}$.

  Write $\vec{x}_1(t)$ as $\vec{u}(t) + i\vec{w}(t)$ and the general solution is $\vec{x}(t) = C_1\vec{u}(t) + C_2\vec{w}(t)$.
- Phase portraits; x_2 vs. x_1. The "eigenvector lines" contain straight-line solutions.

Examples:



$\lambda_1 > \lambda_2 > 0$ $\quad$ $\lambda_1 < \lambda_2 < 0$ $\quad$ $\lambda_1 < 0 < \lambda_2$ $\quad$ $\lambda_1 = a \pm bi$