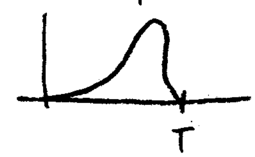
MTHSc 208, Fall 2010

<u>Week 10 Summary:</u>

$f_T(t)$



- If $f(t)$ is <u>periodic</u> with period T and "window"

  then $\mathcal{L}\{f(t)\}(s) = \dfrac{F_T(t)}{1-e^{-sT}} = F_T(s)\left[1+e^{-sT}+e^{-2sT}+e^{-3sT}+\dots\right]$

- <u>Delta function</u>: $\delta_p(t) = \begin{cases} \text{"}\infty\text{"} & t=p \\ 0 & t\neq p \end{cases} = \lim\limits_{\varepsilon\to 0} \delta_p^{\varepsilon}(t)$

  * Not really a function
  * Is "the derivative" of $\mathcal{U}_p(t)$
  * $\displaystyle\int_{-\infty}^{\infty} \delta_p(t)\,dt = 1$, $\displaystyle\int_{-\infty}^{\infty} \delta_p(t)\,f(t)\,dt = f(p)$
  * $\mathcal{L}\{\delta_0(t)\}(s) = 1$, $\mathcal{L}\{\delta_p(t)\}(s) = e^{-sp}$
  * Models a <u>unit impulse force</u>.

- A dot product, or "inner product" allows us to project vectors onto unit vectors.

- For $Per_{2L} = \{$ 2L-periodic functions $\}$, define

  $\langle f(x), g(x)\rangle = \dfrac{1}{L}\displaystyle\int_{-L}^{L} f(x)\,g(x)\,dx$. This allows us to compute the <u>Fourier series</u> of $f(x)$.

- If $f(x)$ is 2π-periodic, then $f(x) = \dfrac{a_0}{2} + \displaystyle\sum_{n=1}^{\infty} a_n\cos nx + b_n \sin nx$

  $a_n = \dfrac{1}{\pi}\displaystyle\int_{-\pi}^{\pi} f(x)\cos nx\,dx$, $b_n = \dfrac{1}{\pi}\displaystyle\int_{-\pi}^{\pi} f(x)\sin nx\,dx$