

Week 11 summary:

• Even functions

* $f(x) = f(-x)$

* Symmetric about y-axis

* Fourier series contains only cosines

* $\int_{-L}^L f(x) dx = 2 \int_0^L f(x) dx$

Odd functions

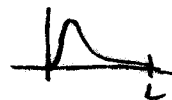
* $f(x) = -f(-x)$

* Symmetric about origin

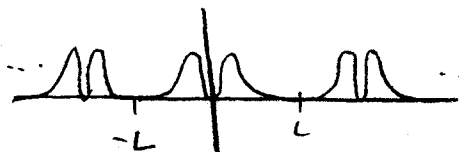
* Fourier series contains only sines

* $\int_{-L}^L f(x) dx = 0$

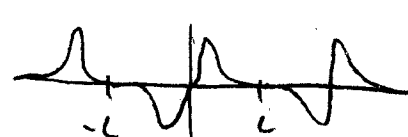
• Fourier cosine & sine series

Start with a function $f(x)$ on $[0, L]$, e.g., 

* Fourier cosine series is the Fourier series of the even extension.

e.g.,  $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$, $a_n = \frac{2}{L} \int_0^L f(x) \cos\left(\frac{n\pi x}{L}\right) dx$

* Fourier sine series is the Fourier series of the odd extension.

e.g.,  $f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$, $b_n = \frac{2}{L} \int_0^L f(x) \sin\left(\frac{n\pi x}{L}\right) dx$

• Complex Fourier series: $f(x) = \sum_{n=-\infty}^{\infty} c_n e^{-inx} = c_0 + \sum_{n=1}^{\infty} (c_n e^{-inx} + c_{-n} e^{inx})$

$c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{-inx} dx$ (for 2π -periodic functions)

$a_n = c_n + c_{-n}$

$b_n = i(c_n - c_{-n})$

$c_n = \frac{a_n - ib_n}{2}$

$c_{-n} = \frac{a_n + ib_n}{2}$