MTHSC 208, Fall 2010

Week 13 summary:

- Solving the heat equation: $u_t = c^2 u_{xx}$

  * Assume $u(x,t) = f(x)\,g(t)$. Compute u_t, u_{xx}, etc.

  * Plug back in and separate variables; set equal to λ.

  * Solve ODE's for $g(t)$ and $f(x)$

  * Get a sol'n $u_n(x,t) = f_n(x) g_n(t)$ for each n.

  * Gen'l sol'n is $u(x,t) = \sum_{n=0}^{\infty} u_n(x,t)$ (superposition)

  * Plug in $t=0$ & use the initial condition (may require finding a Fourier sine or cosine series).

- Boundary conditions:

  * Dirichlet: $u(0,t)=0$, $u(L,t)=0$.
    (temp. of endpoints fixed at $0°$)

  * Neumann: $u_x(0,t)=0$, $u_x(L,t)=0$
    (insulated endpoints; flow of heat is 0).

- Inhomogeneous eq'n: (non-zero boundary conditions)

  First solve the homogeneous eq'n (when BC's are zero).

  Add the steady-state sol'n, $u_{ss}(x)$.

Final solution: $u(x,t) = u_h(x,t) + u_{ss}(x)$.