Consider the system of differential equations:
\[
\begin{align*}
    x_1' &= -x_1 - x_2 \\
    x_2' &= x_1 - 3x_2
\end{align*}
\]

1. Write this in matrix form, \( \mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{b} \).

2. Knowing that \( \mathbf{A} \) has a repeated eigenvalue, \( \lambda_{1,2} = -2 \), and one eigenvector, \( \mathbf{v}_1 = (1, 1) \), write down a solution \( \mathbf{x}_1(t) \) to \( \mathbf{x}' = \mathbf{A}\mathbf{x} \).

3. To find a second solution, assume that \( \mathbf{x}_2(t) = te^{\lambda t}\mathbf{v} + e^{\lambda t}\mathbf{w} \). Plug this back into \( \mathbf{x}' = \mathbf{A}\mathbf{x} \) and equate coefficients (of \( te^{-\lambda t} \) and \( e^{\lambda t} \)) to get a system of two equations, involving \( \mathbf{v} \), \( \mathbf{w} \), and \( \mathbf{A} \).
4. Solve for $v$ by inspection. Plug this back into the second equation and solve for $w$ (it will involve a constant, $C$).

5. Using what you got for $v(t)$ and $w(t)$, write down a solution $x_2(t)$ that is not a scalar multiple of $x_1$. (Pick the simplest value of $C$ that works.)

6. Write down the general solution, $x(t)$.

7. As $t \to \infty$, which of the three terms of $x(t)$ “goes to zero slower”? Use this intuition to sketch the phase portrait.