1. Solve the following differential equations.

   (a) $y' = -3y$

   (b) $2y' = t + 6y$

   (c) $2y' = t^2 + 6y$

   (d) $y'' + 4y = 0$

   (e) $y'' = -9y + 12$.

2. For each system below, write it as $\mathbf{Ax} = \mathbf{b}$. Find all solutions, and sketch the graph of the lines in each system on the same axis. Are the resulting lines intersecting, parallel, or coincident?

   (a) $x_1 + 3x_2 = 0, \quad 2x_1 - x_2 = 0$

   (b) $-x_1 + 2x_2 = 4, \quad 2x_1 - 4x_2 = -6$

   (c) $2x_1 - 3x_2 = 4, \quad x_1 + 2x_2 = -5$

   (d) $3x_1 - 2x_2 = 0, \quad -6x_1 + 4x_2 = 0$

   (e) $2x_1 - 3x_2 = 6, \quad -4x_1 + 6x_2 = -12$

3. For each part, find the determinant, eigenvalues and eigenvectors of the given matrix. If the matrix is invertible, find its inverse.

   (a) $\mathbf{A} = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix}$ (b) $\mathbf{A} = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix}$ (c) $\mathbf{A} = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$

   (d) $\mathbf{A} = \begin{pmatrix} 1 & -2 \\ 3 & -4 \end{pmatrix}$ (e) $\mathbf{A} = \begin{pmatrix} -1 & -4 \\ 1 & -1 \end{pmatrix}$ (f) $\mathbf{A} = \begin{pmatrix} 5/4 & 3/4 \\ -3/4 & -1/4 \end{pmatrix}$

4. For each problem below, find the eigenvalues of the given matrix, and then describe how the nature of the eigenvalue (e.g., positive/negative, complex, repeated, etc.) depends on the parameter α.

   (a) $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & \alpha \end{pmatrix}$ (b) $\mathbf{A} = \begin{pmatrix} 1 & -\alpha \\ 2\alpha & 3 \end{pmatrix}$

5. Let A be a 2×2 matrix. In this problem we will show that $\lambda = 0$ is an eigenvalue of a matrix $\mathbf{A}$ if and only if $\det(\mathbf{A}) = 0$.

   (a) Write the characteristic polynomial (i.e., the polynomial $\det(\mathbf{A} - \lambda I) = 0$ in terms of the determinant and trace of A.

   (b) Show that if $\lambda = 0$ is an eigenvalue of $\mathbf{A}$, then $\det(\mathbf{A}) = 0$.

   (c) Show that if $\det(\mathbf{A}) = 0$, then $\lambda = 0$ is an eigenvalue of $\mathbf{A}$.

   (d) Now, make the same argument – that $\lambda = 0$ is an eigenvalue if and only if $\det(\mathbf{A}) = 0$, without reference to the characteristic polynomial. (*Hint*: If $\lambda = 0$ is an eigenvalue, then $\mathbf{Av} = 0\mathbf{v} = \mathbf{0}$ for some $\mathbf{v} \neq \mathbf{0}$. When does such a homogeneous system have a non-zero solution?)