1. Find the general solution for each of the given system of equations. Draw a phase portrait. Describe the behavior of the solutions as $t \to \infty$.
   
   (a) $\mathbf{x}' = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix} \mathbf{x}$  
   (b) $\mathbf{x}' = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} \mathbf{x}$  
   (c) $\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} \mathbf{x}$  
   (d) $\mathbf{x}' = \begin{pmatrix} 4 & -3 \\ 8 & -6 \end{pmatrix} \mathbf{x}$

2. In each of the next four problems, the eigenvalues and eigenvectors of a matrix $A$ are given. Consider the corresponding system $\mathbf{x}' = A \mathbf{x}$. Without using a computer, draw each of the following graphs.
   
   (i) Sketch a phase portrait of the system.
   (ii) Sketch the solution curve passing through the initial point $(2, 3)$.
   (iii) For the curve in part (ii), sketch the component plots of $x_1$ versus $t$ and $x_2$ versus $t$ on the same set of axes.

   (a) $\lambda_1 = -1, \quad \mathbf{v}_1 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}; \quad \lambda_2 = -4, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.
   (b) $\lambda_1 = 1, \quad \mathbf{v}_1 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}; \quad \lambda_2 = -4, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.
   (c) $\lambda_1 = -1, \quad \mathbf{v}_1 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}; \quad \lambda_2 = 4, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.
   (d) $\lambda_1 = 1, \quad \mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}; \quad \lambda_2 = 4, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$.

3. In each of the next four problems, the eigenvalues and eigenvectors of a matrix $A$ are given. Consider the corresponding system $\mathbf{x}' = A \mathbf{x}$. Without using a computer, draw each of the following graphs.
   
   (i) Sketch a phase portrait of the system.
   (ii) Sketch the trajectory passing through the initial point $(2, 3)$.

   (a) $\lambda_1 = -4, \quad \mathbf{v}_1 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}; \quad \lambda_2 = -1, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.
   (b) $\lambda_1 = 4, \quad \mathbf{v}_1 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}; \quad \lambda_2 = -1, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.
   (c) $\lambda_1 = -4, \quad \mathbf{v}_1 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}; \quad \lambda_2 = 1, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.
   (d) $\lambda_1 = 4, \quad \mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}; \quad \lambda_2 = 1, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$.

4. Find the general solution for each of the given systems in terms of real-valued function, and draw a phase portrait. Describe the behavior of the solutions as $t \to \infty$.

   (a) $\mathbf{x}' = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix} \mathbf{x}$  
   (b) $\mathbf{x}' = \begin{pmatrix} 1 & -1 \\ 5 & -3 \end{pmatrix} \mathbf{x}$  
   (c) $\mathbf{x}' = \begin{pmatrix} 1 & 2 \\ -5 & -1 \end{pmatrix} \mathbf{x}$
5. In the problems below, the coefficient matrix contains a parameter $\alpha$.

(a) Determine the eigenvalues in terms of $\alpha$.

(b) Find the critical value or values of $\alpha$ where the qualitative nature of the phase portrait for the system changes.

(c) Draw a phase portrait for a value of $\alpha$ slight below, and for another value slightly above, each critical value.

(d) Draw a phase portrait when $\alpha$ is exactly the critical value.

\[
\begin{align*}
\text{(a) } \mathbf{x}' &= \begin{pmatrix} \alpha & 1 \\ -1 & \alpha \end{pmatrix} \mathbf{x} \\
\text{(b) } \mathbf{x}' &= \begin{pmatrix} -1 & \alpha \\ -1 & -1 \end{pmatrix} \mathbf{x}
\end{align*}
\]

6. Find the general solution for each of the given systems and draw a phase portrait. Describe the behavior of the solutions as $t \to \infty$.

\[
\begin{align*}
\text{(a) } \mathbf{x}' &= \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} \mathbf{x} \\
\text{(b) } \mathbf{x}' &= \begin{pmatrix} -3/2 & 1 \\ -1/4 & -1/2 \end{pmatrix} \mathbf{x} \\
\text{(c) } \mathbf{x}' &= \begin{pmatrix} -1 & -1/2 \\ 2 & -3 \end{pmatrix} \mathbf{x}
\end{align*}
\]