1. Solve the following differential equations:
   (a) $y'' + 6y' + 9y = 5$
   (b) $y'' = -\omega^2 y$
   (c) $y' + 2y = e^t$
   (d) $y' + 3y = 0$.

2. Find the Laplace transform of the following functions by explicitly computing $\displaystyle\int_0^\infty f(t)\, e^{-st}\, dt$.
   (a) $f(t) = 3$
   (b) $f(t) = e^{3t}$
   (c) $f(t) = \cos 2t$
   (d) $f(t) = te^{2t}$
   (e) $f(t) = e^{-3t} \sin 2t$

3. Sketch each of the following piecewise defined functions, and compute their Laplace transforms.
   (a) $f(t) = \begin{cases} 0, & 0 \leq t < 4 \\ 5, & t \geq 4 \end{cases}$ (b) $f(t) = \begin{cases} t, & 0 \leq t < 3 \\ 3, & t \geq 3 \end{cases}$

4. Engineers frequently use the *Heavyside function*, defined by
$$H(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$
   to emulate turning on a switch at a certain instance in time. Sketch the graph of the function $x(t) = e^{0.2t}$ and compute its Laplace transform, $X(s)$. On a different set of axes, sketch the graph of
$$y(t) = H(t-3)e^{0.2t}$$
   and calculate its Laplace transform, $Y(s)$. How do $X(s)$ and $Y(s)$ differ? What do you think the Laplace transform of $H(t-c)e^{0.2t}$ is, where c is an arbitrary positive constant?

5. Find the Laplace transform of the following functions by using a table of Laplace transforms
   (a) $f(t) = -2$
   (b) $f(t) = e^{-2t}$
   (c) $f(t) = \sin 3t$
   (d) $f(t) = te^{-3t}$
   (e) $f(t) = e^{2t} \cos 2t$

6. Transform the given initial value problem into an algebraic equation involving $Y(s) := \mathcal{L}(y)$, and solve for $Y(s)$.

   (a) $y'' + y = \sin 4t, \quad y(0) = 0, \quad y'(0) = 1$

   (b) $y'' + y' + 2y = \cos 2t + \sin 3t, \quad y(0) = -1, \quad y'(0) = 1$

   (c) $y' + y = e^{-t} \sin 3t, \quad y(0) = 0$