1. Solve the following differential equations:
   
   (a) \( y'' + 6y' + 9y = 5 \)
   (b) \( y'' = -\omega^2 y \)
   (c) \( y' + 2y = e^t \)
   (d) \( y' + 3y = 0 \).

2. Find the Laplace transform of the following functions by explicitly computing \( \int_0^\infty f(t) e^{-st} \, dt \).
   
   (a) \( f(t) = 3 \)
   (b) \( f(t) = e^{3t} \)
   (c) \( f(t) = \cos 2t \)
   (d) \( f(t) = te^{2t} \)
   (e) \( f(t) = e^{-3t} \sin 2t \)

3. Sketch each of the following piecewise defined functions, and compute their Laplace transforms.
   
   (a) \( f(t) = \begin{cases} 
   0, & 0 \leq t < 4 \\
   5, & t \geq 4 
   \end{cases} \)
   (b) \( f(t) = \begin{cases} 
   t, & 0 \leq t < 3 \\
   3, & t \geq 3 
   \end{cases} \)

4. Engineers frequently use the *Heavyside function*, defined by

\[
H(t) = \begin{cases} 
0, & t < 0 \\
1, & t \geq 0 
\end{cases}
\]

   to emulate turning on a switch at a certain instance in time. Sketch the graph of the function \( x(t) = e^{0.2t} \) and compute its Laplace transform, \( X(s) \). On a different set of axes, sketch the graph of

\[
y(t) = H(t - 3)e^{0.2t}
\]

   and calculate its Laplace transform, \( Y(s) \). How do \( X(s) \) and \( Y(s) \) differ? What do you think the Laplace transform of \( H(t - c)e^{0.2t} \) is, where \( c \) is an arbitrary positive constant?

5. Find the Laplace transform of the following functions by using a table of Laplace transforms

   (a) \( f(t) = -2 \)
   (b) \( f(t) = e^{-2t} \)
   (c) \( f(t) = \sin 3t \)
   (d) \( f(t) = te^{-3t} \)
   (e) \( f(t) = e^{2t} \cos 2t \)
6. Transform the given initial value problem into an algebraic equation involving \( Y(s) := \mathcal{L}(y) \), and solve for \( Y(s) \).

(a) \( y'' + y = \sin 4t, \ y(0) = 0, \ y'(0) = 1 \)

(b) \( y'' + y' + 2y = \cos 2t + \sin 3t, \ y(0) = -1, \ y'(0) = 1 \)

(c) \( y' + y = e^{-t} \sin 3t, \ y(0) = 0 \)