1. Find the inverse Laplace transform of the following functions.

(a) \( Y(s) = \frac{2}{3 - 5s} \)

(b) \( Y(s) = \frac{1}{s^2 + 4} \)

(c) \( Y(s) = \frac{5s}{s^2 + 9} \)

(d) \( Y(s) = \frac{3}{s^2} \)

(e) \( Y(s) = \frac{3s + 2}{s^2 + 25} \)

(f) \( Y(s) = \frac{2 - 5s}{s^2 + 9} \)

(g) \( Y(s) = \frac{s}{(s + 2)^2 + 4} \)

(h) \( Y(s) = \frac{3s + 2}{s^2 + 4s + 29} \)

(i) \( Y(s) = \frac{2s - 2}{(s - 4)(s + 2)} \)

(j) \( Y(s) = \frac{3s^2 + s + 1}{(s - 2)(s^2 + 1)} \)

2. Use the Laplace transform to solve the following initial value problems.

(a) \( y' - 4y = e^{-2t^2}, \; y(0) = 1 \)

(b) \( y'' - 9y = -2e^t, \; y(0) = 0, \; y'(0) = 1 \)

3. Find the Laplace transform of the given functions.

(a) \( 3H(t - 2) \)

(b) \( (t - 2)H(t - 2) \)

(c) \( e^{2(t-1)}H(t - 1) \)

(d) \( H(t - \pi/4)\sin 3(t - \pi/4) \)

(e) \( t^2H(t - 1) \)

(f) \( e^{-t}H(t - 2) \)

4. In this exercise, you will examine the effect of shifts in the time domain on the Laplace transform (graphically).

(a) Sketch the graph of \( f(t) = \sin t \) in the time domain. Find the Laplace transform \( F(s) = \mathcal{L}\{f(t)\}(s) \). Sketch the graph of \( F \) in the s-domain on the interval \([0, 2]\).
(b) Sketch the graph of \( g(t) = H(t-1) \sin(t-1) \) in the time domain. Find the Laplace transform \( G(s) = \mathcal{L}\{g(t)\}(s) \). Sketch the graph of \( G \) in the \( s \)-domain on the interval \([0, 2]\) on the same axes used to sketch the graph of \( F \).

(c) Repeat the directions in part (b) for \( g(t) = H(t-2) \sin(t-2) \). Explain why engineers like to say that “a shift in the time domain leads to an attenuation (scaling) in the \( s \)-domain.”

5. Use the Heaviside function to concisely write each piecewise function.

   (a) \( f(t) = \begin{cases} 
   5 & 2 \leq t < 4; \\
   0 & \text{otherwise} 
   \end{cases} \)

   (b) \( f(t) = \begin{cases} 
   0 & t < 0; \\
   t & 0 \leq t < 3 \\
   4 & t \geq 3 
   \end{cases} \)

   (c) \( f(t) = \begin{cases} 
   0 & t < 0; \\
   t^2 & 0 \leq t < 2 \\
   4 & t \geq 2 
   \end{cases} \)

6. Find the inverse Laplace transform of each function. Create a piecewise definition for your solution that doesn’t use the Heavyside function.

   (a) \( F(s) = \frac{e^{-2s}}{s + 3} \)

   (b) \( F(s) = \frac{1 - e^{-s}}{s^2} \)

   (c) \( F(s) = \frac{e^{-s}}{s^2 + 4} \)