

1. Find the inverse Laplace transform of the following functions.

(a) $Y(s) = \frac{2}{3 - 5s}$

(b) $Y(s) = \frac{1}{s^2 + 4}$

(c) $Y(s) = \frac{5s}{s^2 + 9}$

(d) $Y(s) = \frac{3}{s^2}$

(e) $Y(s) = \frac{3s + 2}{s^2 + 25}$

(f) $Y(s) = \frac{2 - 5s}{s^2 + 9}$

(g) $Y(s) = \frac{s}{(s + 2)^2 + 4}$

(h) $Y(s) = \frac{3s + 2}{s^2 + 4s + 29}$

(i) $Y(s) = \frac{2s - 2}{(s - 4)(s + 2)}$

(j) $Y(s) = \frac{3s^2 + s + 1}{(s - 2)(s^2 + 1)}$

2. Use the Laplace transform to solve the following initial value problems.

(a) $y' - 4y = e^{-2t}t^2, \quad y(0) = 1$

(b) $y'' - 9y = -2e^t, \quad y(0) = 0, \quad y'(0) = 1$

3. Find the Laplace transform of the given functions.

(a) $3H(t - 2)$

(b) $(t - 2)H(t - 2)$

(c) $e^{2(t-1)}H(t - 1)$

(d) $H(t - \pi/4) \sin 3(t - \pi/4)$

(e) $t^2H(t - 1)$

(f) $e^{-t}H(t - 2)$

4. In this exercise, you will examine the effect of shifts in the time domain on the Laplace transform (graphically).

(a) Sketch the graph of $f(t) = \sin t$ in the time domain. Find the Laplace transform $F(s) = \mathcal{L}\{f(t)\}(s)$. Sketch the graph of F in the s -domain on the interval $[0, 2]$.

- (b) Sketch the graph of $g(t) = H(t - 1) \sin(t - 1)$ in the time domain. Find the Laplace transform $G(s) = \mathcal{L}\{g(t)\}(s)$. Sketch the graph of G in the s -domain on the interval $[0, 2]$ on the same axes used to sketch the graph of F .
- (c) Repeat the directions in part (b) for $g(t) = H(t - 2) \sin(t - 2)$. Explain why engineers like to say that “a shift in the time domain leads to an attenuation (scaling) in the s -domain.”
5. Use the Heaviside function to concisely write each piecewise function.
- (a) $f(t) = \begin{cases} 5 & 2 \leq t < 4; \\ 0 & \text{otherwise} \end{cases}$
- (b) $f(t) = \begin{cases} 0 & t < 0; \\ t & 0 \leq t < 3 \\ 4 & t \geq 3 \end{cases}$
- (c) $f(t) = \begin{cases} 0 & t < 0; \\ t^2 & 0 \leq t < 2 \\ 4 & t \geq 2 \end{cases}$
6. Find the inverse Laplace transform of each function. Create a piecewise definition for your solution that doesn't use the Heaviside function.

(a) $F(s) = \frac{e^{-2s}}{s + 3}$

(b) $F(s) = \frac{1 - e^{-s}}{s^2}$

(c) $F(s) = \frac{e^{-s}}{s^2 + 4}$