

1. Solve the following differential equations:

(a) $y' - 2y = 0$

(b) $y' - 2y = t - 3$

(c) $y' - 2y = e^{3t}$

(d) $y'' - 4y = 0$

(e) $y'' + 4y = 0$

(f) $y'' + 4y' + 3y = 10$.

2. The function

$$f(x) = \begin{cases} 0 & -\pi \leq x < -\pi/2, \\ 1 & -\pi/2 \leq x < \pi/2, \\ 0 & \pi/2 \leq x \leq \pi, \end{cases}$$

can be extended to be periodic of period 2π . Sketch the graph of the resulting function, and compute its Fourier series.

3. The function

$$f(t) = |x|, \quad \text{for } x \in [-\pi, \pi]$$

can be extended to be periodic of period 2π . Sketch the graph of the resulting function, and compute its Fourier series.

4. The function

$$f(x) = \begin{cases} 0 & -\pi \leq x < 0, \\ x & 0 \leq x \leq \pi, \end{cases}$$

can be extended to be periodic of period 2π . Sketch the graph of the resulting function, and compute its Fourier series.

5. Consider the 2π -periodic function defined by

$$f(x) = \begin{cases} x^2 & -\pi \leq x < \pi, \\ f(x - 2k\pi), & -\pi + 2k\pi \leq x < \pi + 2k\pi. \end{cases}$$

Sketch this function (at least for $k = -2, -1, 0, 1, 2$) and compute its Fourier series.

6. Find the Fourier series of the following functions *without* computing any integrals.

(a) $f(x) = 2 - 3 \sin 4x + 5 \cos 6x$,

(b) $f(x) = \sin^2 x$ [*Hint*: Use a standard trig identity.]

7. Determine which of the following functions are even, which are odd, and which are neither even nor odd:

(a) $f(t) = x^3 + 3x$.

(b) $f(t) = x^2 + |x|$.

(c) $f(t) = e^x$.

(d) $f(t) = \frac{1}{2}(e^x + e^{-x})$.

(e) $f(t) = \frac{1}{2}(e^x - e^{-x})$.