$\boxed{\text{MTHSc 208, Fall 2011}}$

Week 7 summary:

- A matrix $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ has an inverse iff $\det A := a_{11}a_{22} - a_{12}a_{21} \neq 0$.

  The inverse of A is $A^{-1} := \frac{1}{\det A} \begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix}$, and

  $AA^{-1} = A^{-1}A = I := \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

  The solution to the system $A\vec{x} = \vec{b}$ is $\vec{x} = A^{-1}\vec{b}$.

- If $A\vec{v} = \lambda\vec{v}$, then $\vec{v}$ is an eigenvector of A with eigenvalue λ.

  To find λ, solve $\det(A - \lambda I) = 0$ for λ.

  To find v, solve $(A - \lambda I)\vec{v}$ for $\vec{v}$.

  Note: $\det(A - \lambda I) = \lambda^2 - (\mathrm{tr}\, A)\lambda + (\det A) = 0$, $\mathrm{tr}\, A := a_{11} + a_{22}$.

- The system $\vec{x}' = A\vec{x}$ has general solution $\vec{x}(t) = C_1 e^{\lambda_1 t}\vec{v}_1 + C_2 e^{\lambda_2 t}\vec{v}_2$, where $\lambda_{1,2}$ are the eigenvalues of A, and $v_{1,2}$ are the eigenvectors.

- To solve $\vec{x}' = A\vec{x} + \vec{b}$, find the steady-state solution $\vec{x}_{ss}(t)$ (set $\vec{x}' = 0$), then solve the homogeneous system (set $\vec{b} = 0$).

  The general solution is $\vec{x}(t) = \vec{x}_h(t) + \vec{x}_{ss}(t)$.