Week 14 & 15 summary:

- **Partial differential equation (PDE)**: Equations involving a multivariate function and its partial derivatives.

- **Heat equation**: \( u_t = c^2 u_{xx} \), \( u(x,t) \) = temp. at pos. \( x \), time \( t \).

  - **Boundary conditions**: e.g., \( u(0,t) = u(L,t) = 0 \)
  - **Initial condition**: e.g., \( u(x,0) = f(x) \)

- **Solving the heat equation**: \( u_t = c^2 u_{xx} \)
  - Assume \( u(x,t) = f(x)g(t) \). Compute \( u_t, u_{xx} \), "zero-boundary condition".
  - Plug back in and separate variables, set equal to \( \lambda \).
  - Solve ODE's for \( g(t) \) and \( f(x) \), and determine \( \lambda \).
  - Get a sol'n \( u_n(x,t) = f_n(x)g_n(t) \) for each \( n \).
  - Gen'l sol'n is \( u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) \) (superposition)
  - Plug in \( t=0 \) & use initial condition (may require finding a Fourier sine or cosine series).

- **Boundary conditions** for the heat equation:
  - **Dirichlet**: specify the value, e.g., \( u(0,t) = T_1 \), \( u(L,t) = T_2 \)
  - **von Neumann**: specify the derivative, e.g., \( u_x(0,t) = 0 \), \( u_x(L,t) = 0 \).

  This represents insulated endpoints.

If boundary conditions are non-zero: \( u(x,t) = u_h(x,t) + u_{ss}(x) \).
Wave equation: \( u_{tt} = c^2 u_{xx} \)

Boundary conditions: \( u(0, t) = u(L, t) = 0 \)

Initial conditions: \( u(x, 0) = h_1(x) \) "initial position"
\( u_t(x, 0) = h_2(x) \) "initial velocity"

Main difference: \( g(t) = a \cos(c x t) + b \sin(c x t) \) instead of \( A e^{-c^2 x t} \)