1. *Due Friday 9/2.* Fix $\lambda \geq 2$, and let $\mathsf{T}(n)$ denote the number of RNA secondary structures with arc length at least $\lambda$ over $[n]$. Via the bijection between secondary structures and Motzkin paths, we have a recursion:

$$\mathsf{T}(n) = \mathsf{T}(n-1) + \sum_{j=0}^{n-(\lambda+1)} \mathsf{T}(n-2-j)\mathsf{T}(j)\,.$$

Define the following generating function

$$\mathbf{T}(z) = \sum_{n=0}^{\infty} \mathsf{T}(n) z^n\,.$$

The textbook says that if we multiply the recursion by $z^n$ for all $n > \lambda$, then subsequent calculation gives the equation

$$z^2\mathbf{T}(z)^2 - (1 - z + z^2 + \cdots + z^\lambda)\mathbf{T}(z) + 1 = 0\,.$$

Carry out the details of this derivation.

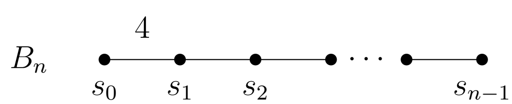
2. *Due Wednesday 9/7.* Start with the empty Young tableau, and apply the RSK-algorithm to the following sequence: $2, 7, 5, 8, 3, 4, 6, 1$. Draw each intermediate tableau.

3. *Due Wednesday 9/7.* Consider the following Young tableau $T_i$ over the set $\{0, 1, \ldots, 9\}$:

| | | | |
|---|---|---|---|
| 0 | 2 | 5 | 7 |
| 1 | 3 | 9 | |
| 4 | 6 | | |
| 8 | | | |

Find all standard Young tableaux $T_{i-1}$ such that applying the RSK-algorithm to $T_{i-1}$ yields $T_i$.

4. *Due Monday 9/12.* Let $\mathsf{B}_n$ be the Weyl group with the following Coxeter graph:




Recall that this group is isomorphic to the group of *signed permutations* of a stack of $n$ cards numbered $\{1, \ldots, n\}$, each with a black front, and a white back. The canonical isomorphism sends

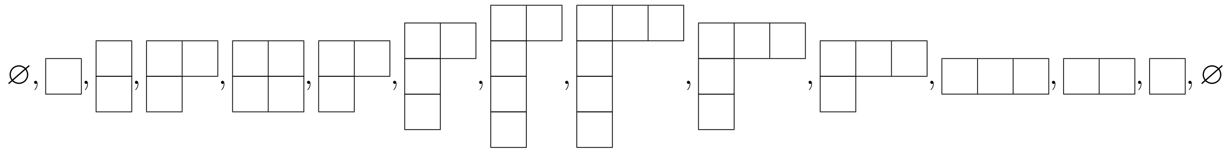
$$s_i \longmapsto (i \;\; i+1) \qquad\qquad s_0 \longmapsto \text{“flip over top card”},$$

where $(i \;\; i+i)$ means transpose the two cards in positions $i$ and $i+1$. Consider the subgroup consisting of all signed permutations with the property that only an even number of cards are flipped (i.e., white side showing). This is generated by the set $\{s_0', s_1, \ldots, s_{n-1}\}$, where

$$s_i \longmapsto (i \;\; i+1) \qquad\qquad s_0' \longmapsto \text{“flip over \textit{and} swap top 2 cards”}.$$

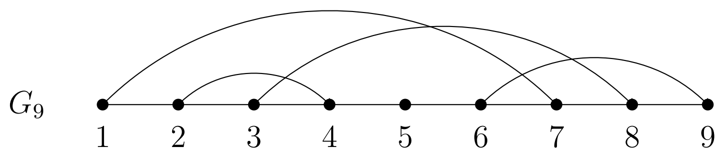
Write $s_0'$ in terms of $s_0, s_1, \ldots, s_n$. Draw the Coxeter graph for this group. How many elements does it contain?

5. *Due Monday 9/19.* Consider the following $*$-tableaux $(\mu^i)_{i=0}^{14}$:



Construct the corresponding arc-diagram $\psi((\mu^i)_{i=0}^{14})$.

6. *Due Monday 9/19.* Consider the following arc diagram:




Construct the corresponding $*$-tableaux, $(\mu^i)_{i=0}^{9} = \psi^{-1}(G_9)$.

7. *Due Monday 9/26.* Let $f_k(n,0)$ be the number of $k$-noncrossing matchings without isolated vertices over $[n]$. Use the residue theorem to show that

$$f_k(2m,0) = \frac{1}{2\pi i}\oint_{|v|=\beta} \mathbf{F}_k(v^2)v^{-2m-1}\,dv\,,$$

where $\beta > 0$ and $\mathbf{F}_k(z) = \sum_{n=0}^{\infty} f_k(2n,0)z^n$.

8. *Due Friday 10/21.* Verify that the recurrence

$$(m+1)g_k(s+1,m+1) = (m+1)g_k(s,m+1) + (2s+1-m)g_k(s,m)$$

is "equivalent" (and explain what you mean by this) to the partial differential equation

$$\frac{\partial \mathbf{G}_k(x,y)}{\partial y} = x\frac{\partial \mathbf{G}_k(x,y)}{\partial y} + 2x^2\frac{\partial \mathbf{G}_k(x,y)}{\partial x} + x\mathbf{G}_k(x,y) - xy\frac{\partial \mathbf{G}_k(x,y)}{\partial y}.$$

9. *Due Friday 11/4.* Let $\mathsf{T}_k(n,h)$ and $\mathsf{C}_k(n,h)$ be the number of $k$-noncrossing RNA structures and cores, respectively, over $[n]$ with $h$ arcs. Define the functions

$$a(i) = \mathsf{C}_k(n-2(h-1-i), i+1), \qquad b(i) = \mathsf{T}_{k-1}(n-2(h-1-i), i+1)\,,$$

for $i = 0, 1, \ldots, h-1$. The Core Lemma tells us that

$$b(h-1) = \sum_{i=0}^{h-1}\binom{h-1}{i}a(i)\,.$$

The textbook says that *Möbius inversion* can be used conclude that

$$a(h-1) = \sum_{i=0}^{h-1}(-1)^{h-1-i}\binom{h-1}{i}b(i)\,.$$

Carry out the details in this derivation.

10. *Due Monday 11/21.* Prove the lemma from class showing that the vertex coloring part of the Loop Decomposition Theorem is well-defined:

    Let $S$ be an arc diagram with the edges colored via the algorithm described in Proposition 6.2. For a given vertex $i$ of the core structure $c(S)$, let $N(i)$ be the $c(S)$-arcs that nest it. Prove that $N(i)$ contains at most one non-red $\prec$-minimal arc.