MthSc 985: Topics in Discrete Mathematical Biology
Midterm
October 31, 2011

NAME:

Instructions

- Exam time is 50 minutes
- Open notes / book / everything.
- **Show your work.** Partial credit will be given.

Question	Points Earned	Maximum Points
1		20
2		20
3		20
4		20
5		20
Total		**100**

1. (a) In your own words, give a concise combinatorial definition of an *RNA secondary structure* and an *RNA pseudoknot structure* of length n.

   (b) For both secondary and pseudoknot structures, there are additional restrictions on e.g., arc length and stack size, that lead to the notion of a *canonical* structure. Give these additional requirements, as well as biophysical justifications for why we impose them.

2. (a) Draw an RNA strand that exhibits a pseudoknot structure, as a 5-noncrossing (but *not* 4-noncrossing) arc diagram. Draw the diagram and an actual RNA strand that it represents.

   (b) Recall the canonical bijection ψ from $*$-tableaux to arc diagrams. Compute $\psi^{-1}(G_n)$, where G_n is your diagram from Part (a).

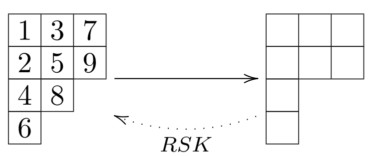
3. Let $\delta \in \mathcal{T}_{k,\sigma}(n)$ be a k-noncrossing, σ-canonical RNA structure.

   (a) Define a *hairpin loop* of δ purely combinatorially. Assume that the endpoints of the loop are i and j, with $i < j$.

   (b) Suppose that δ has shape $\gamma \in \mathcal{I}_k(s, m)$ (that is, s arcs, of which m are 1-arcs). What can you say about the number of hairpin loops that δ has? Justify your answer.

4. (a) Starting with the empty Young tableau, apply the RSK-algorithm to the following sequence: $9, 1, 4, 8, 5, 2, 6, 3, 7$. Draw the resulting Young tableau.

(b) Find the unique Young tableau T_i (at right) with the given shape such that inserting a number into T_i via RSK gives T_{i-1} (at left).



5. In class, we constructed the combinatorial class $\mathcal{T}_\gamma$ of all k-noncrossing, σ-canonical RNA structures (that is, minimum arc length $\lambda = 2$) that have shape $\gamma \in \mathcal{I}_k(s,m)$:

$$\mathcal{T}_\gamma = [\mathcal{K}_\sigma \times \mathrm{Seq}(\mathcal{N}_\sigma)] \times [\mathcal{L}^{2s+1-m} \times (\mathcal{Z} \times \mathcal{L})^m] \, .$$

   Here, $\mathcal{Z}$ is the class of vertices, $\mathcal{L}$ is the class of vertex sequences, $\mathcal{K}_\sigma$ is the class of stacks, and $\mathcal{N}_\sigma$ the class of induced stacks.

   Modify this construction to create the combinatorial class $\mathcal{F}_\gamma$ of all k-noncrossing, σ-canonical diagrams that have shape $\gamma \in \mathcal{I}_k(s,m)$. (That is, drop the minimum arc length $\lambda = 2$ requirement.)