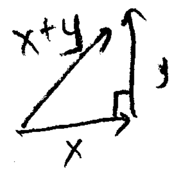
Part II

1 Orthogonality



Vectors x, y are <u>orthogonal</u> if $x^T y = 0$ (i.e., $x \cdot y = 0$).

Recall that $\|x\|^2 = x^T x$.

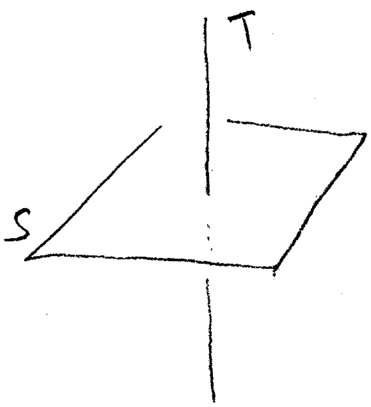
<u>Ex</u>: $x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $y = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$, $z = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$ are orthogonal.

<u>Fact</u>: If $x \perp y$, then $\|x\|^2 + \|y\|^2 = \|x+y\|^2$.

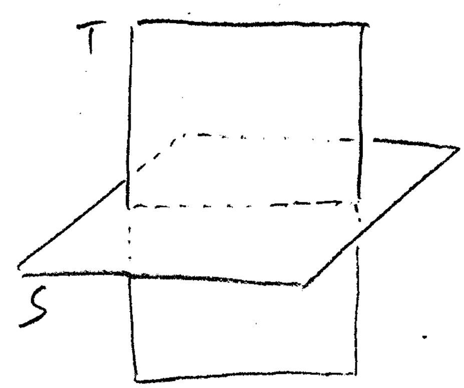
<u>Why</u>: $$\|x+y\|^2 = (x+y)^T(x+y) = x^Tx + y^Ty + \underbrace{x^Ty}_{=0} + \underbrace{y^Tx}_{=0}$$
$$= \|x\|^2 + \|y\|^2.$$

<u>Def</u>: Two subspaces S & T are <u>orthogonal</u> if $s \perp t$ for all $s \in S$, $t \in T$.

<u>Note</u>: If $S \perp T$, then $S \cap T = \{\vec{0}\}$.



yes



no

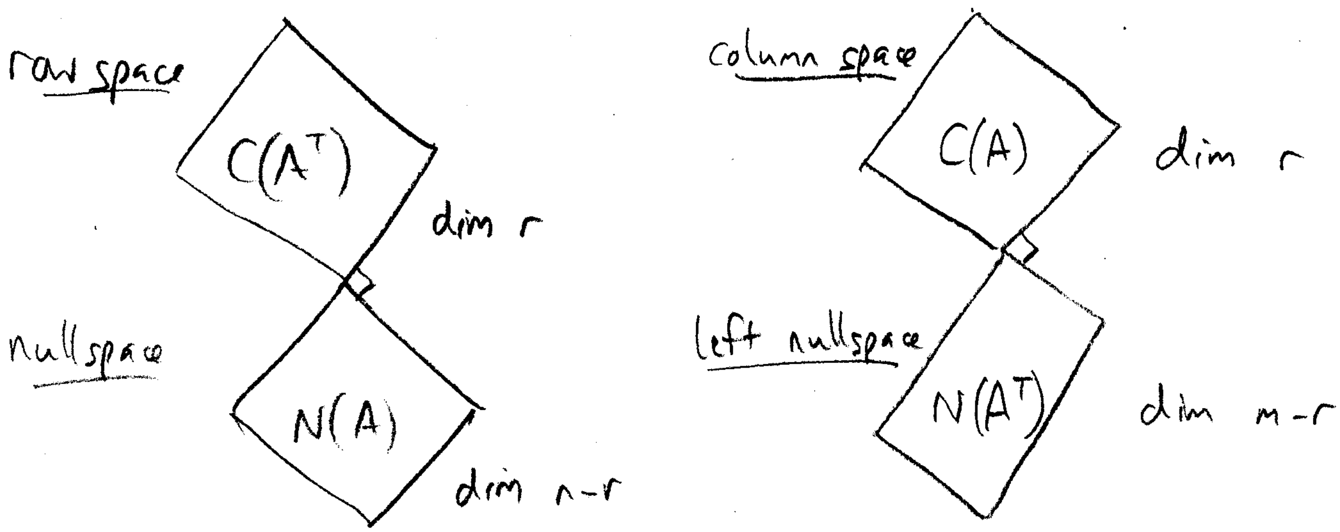
Fact: $\overset{C(A^T)}{\boxed{\text{Row space}}}$ is orthogonal to $\overset{N(A)}{\boxed{\text{nullspace}}}$.

Why: $x \in N(A) \Rightarrow Ax = 0$

$$\begin{bmatrix} (\text{row } 1)^T \\ (\text{row } 2)^T \\ \vdots \\ (\text{row } m)^T \end{bmatrix} \begin{bmatrix} \\ x \\ \\ \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

$$\Rightarrow (\text{row } i)^T x = 0 \quad \text{for each } i.$$

Updated picture:



Ex: $Ax = \begin{bmatrix} 1 & 2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$ $\qquad n = 3$, $r = 1$ $\qquad \perp$ to $\begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$

$N(A)$ is a plane in $\mathbb{R}^3$ (the plane $x + 2y + 5z = 0$)

* Nullspace & rowspace are orthogonal complements in $\mathbb{R}^n$.

contains all vectors $\perp$ to rowspace.