Throughout, a metric space X is endowed with a distance function $d\colon X \times X \to \mathbb{R}$, and a topological space X is endowed with a topology τ, even if not explicitly mentioned.

1. A subset A of a topological space (X, τ) is said to be *dense* in X if $\overline{A} = X$. Prove that if for each open set U, we have $A \cap U \neq \emptyset$, then A is dense in X.

2. Let A be a subset of a topological space (X, τ). The *boundary* of A is the set $\partial A := \overline{A} \cap \overline{A^c}$.
   (a) Prove that $\partial A = \emptyset$ if and only if A is open and closed.
   (b) Prove that $\overline{A} = A \cup \partial A$.
   (c) Prove that A is closed if and only if $\partial A \subseteq A$.
   (d) Prove that A is open if and only if $\partial A \subseteq A^c$.

3. A subset E of a topological space X is called a G_δ if there is a sequence $U_1, U_2, \dots$ of open sets such that $E = \displaystyle\bigcap_{i=1}^{\infty} U_i$.
   (a) Show that if f is a continuous function from X to the real line with the usual topology, then the set $Z = \{x \in X : f(x) = 0\}$ is closed and is a G_δ.
   (b) Show that in a metric space, every closed set is a G_δ.
   (c) Show that the conclusion of Part (b) fails in general for topological spaces.