Throughout, a topological space X is endowed with a topology τ, even if not explicitly mentioned.

1. If $A \times B$ is a compact subset of a product space $X \times Y$ contained in an open set $W \subseteq X \times Y$, prove that there are open sets $U \subseteq X$ and $V \subseteq Y$ such that $A \times B \subseteq U \times V \subseteq W$.

2. Let A, B be disjoint compact subspaces of a Hausdorff space X. Prove that there are disjoint open sets U, V with $A \subseteq U$ and $B \subseteq V$.

3. Suppose C_n is a compact connected subspace of a Hausdorff space X and $C_n \supseteq C_{n+1}$ for each $n \in \mathbb{N}$. Prove that $\displaystyle\bigcap_{n=1}^{\infty} C_n$ is connected.