Throughout, a topological space X is endowed with a topology τ, even if not explicitly mentioned.

1. A collection $\{A_\alpha\}$ of subsets of X satisfies the *finite intersection property* if $\bigcap_{i=1}^{n} A_{\alpha_i} \neq \emptyset$ for any finite subcollection.
   (a) Prove Cantor's "finite intersection lemma": Suppose $\{K_\alpha\}$ is a collection of compact sets of a Hausdorff space X. If $\bigcap_{i=1}^{n} K_{\alpha_i} \neq \emptyset$ for any finite subcollection, then $\bigcap_{\alpha} K_\alpha \neq \emptyset$.
   (b) Prove that X is compact if and only if every collection of closed sets $\{F_\alpha\}$ satisfying the finite intersection property must also satify $\bigcap_{\alpha \in I} F_\alpha \neq \emptyset$.

2. On HW 2, you proved that if a function $f\colon X \to Y$ between metric spaces is continuous, then its graph
$$\Gamma_f := \{(x, f(x)) \mid x \in X\}$$
is a closed subset of $X \times Y$. Now, suppose $f\colon X \to Y$ is a map between topological spaces, and Y is Hausdorff.
   (a) Show that if f is continuous, then the graph Γ_f is closed in $X \times Y$.
   (b) Show that the conclusion of Part (a) may fail of Y is not Hausdorff.
   (c) Show that if X and Y are both compact and Hausdorff, then the converse to Part (a) holds.

3. Let $f\colon X \to Y$ be a continuous mapping of a compact space X onto a Hausdorff space Y. Prove that f is a closed map, and hence a quotient map.

4. Suppose X is a Hausdorff space and $q : X \to Y$ is a quotient map. Further suppose that q is a closed map and that $q^{-1}(y)$ is compact for all $y \in Y$. Prove that Y is Hausdorff.