Throughout, a topological space X is endowed with a topology τ, even if not explicitly mentioned.

1. Show that if $f, f' \colon X \to Y$ are homotopic maps (not necessarily paths) and $g, g' \colon Y \to Z$ are homotopic, then $g \circ f$ and $g' \circ f'$ are homotopic.

2. Given spaces X and Y, let $[X, Y]$ denote the set of homotopy classes of maps of X into Y. A space X is *contractible* if the identity map $i_X \colon X \to X$ is nullhomotopic.

   (a) Show that I and $\mathbb{R}$ are contractible.

   (b) Show that a contractiable space is path connected.

   (c) Show that if Y is contractible, then for any X, the set $[X, Y]$ has a single element.

   (d) Show that if X is contractible and Y is path connected, then $[X, Y]$ has a single element.

3. Let $f \colon I \to X$ be a path with $f(0) = x_0$ and $f(1) = x_1$. Denote its path homotopy class by $[f]$, and let $\bar{f} \colon I \to X$ be the reverse path: $\bar{f}(s) = f(1 - s)$. Let $e_{x_1} \colon I \to X$ denote the constant path at x_1.

   (a) Show that $[f] * [e_{x_1}] = [f]$.

   (b) Show that $[\bar{f}] * [f] = [e_{x_1}]$.