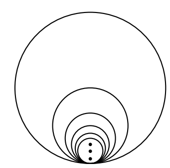
1. Let $p\colon E \to B$ be a convering map of a connected space.
   (a) Show that if $p^{-1}(b_0)$ has k elements for some $b_0 \in B$, then $p^{-1}(b)$ has k elements for every $b \in B$. Such a covering is called a *k-fold covering* of B.
   (b) Show by example how this can fail if B is disconnected.

2. Up to homeomorphism, there are three 2-fold covers of $S^1 \vee S^1$ and seven 3-fold covers. Find them.

3. Let $p\colon E \to B$ be a covering space with $p^{-1}(b)$ finite and nonempty for all $b \in B$. Show that E is compact and Hausdorff iff X is compact and Hausdorff.

4. The *Hawaiian earring* is the subspace H of $\mathbb{C}$ defined as follows:

$$H = \bigcup_{n=1}^{\infty} C_n\,, \qquad C_n = \left\{ \tfrac{1}{n}(1 + e^{2\pi i z}) : z \in \mathbb{C} \right\}$$



Let $z_0 = 0 \in \mathbb{C}$ be the basepoint. This space retracts onto the n^{th} circle C_n via the map

$$r_n\colon H \longrightarrow C_n\,, \qquad r_n(z) = \begin{cases} z & z \in C_n \\ 0 & z \notin C_n. \end{cases}$$

   (a) Prove that the induced homomorphism $(r_n)_*\colon \pi_1(H, z_0) \to \pi_1(C_n, z_0)$ is onto.
   (b) Assuming that $\pi(C_n, z_0) \cong \mathbb{Z}$, prove that $\pi_1(H, z_0)$ is uncountable. [*Hint*: For each binary sequence (b_n), define an element of $\pi_1(H, z_0)$ which goes b_n times around C_n.]