1. Show that there is no retraction from:

   (a) the solid torus $S^1 \times D^2$ to its boundary torus $S^1 \times S^1$;

   (b) the annulus $S^1 \times I$ to its boundary;

   (c) the Möbius band to its boundary.

2. Recall that a space X is contractible if the identity map on X is nullhomotopic. Show that X is contractible if and only if X has the homotopy type of a one-point space.

3. Let S be a set. A free group on S is a group F together with a map $i\colon S \to F$ such that for any other map $j\colon S \to G$ to a group G, there is a unique homomorphism $f\colon F \to G$ such that $j = f \circ i$:

$$\begin{tikzcd} S \arrow[r, "i"] \arrow[dr, "j"'] & F \arrow[d, dashed, "f"] \\ & G \end{tikzcd}$$

   (i) Show that i must be injective.

   (ii) Show that if F and F' are free groups on S, then $F \cong F'$.