

Math 2080: Differential Equations

Worksheet 3.5: Damped harmonic motion

NAME:

1. For the following exercises, rewrite the given function in the form

$$y = A \cos(\omega t - \phi) = A \cos\left(\omega\left(t - \frac{\phi}{\omega}\right)\right),$$

and then plot the graph of this function.

(a) $y = 3 \cos 2t + 4 \sin 2t$

(b) $y = 3 \cos 2t - 4 \sin 2t$

(c) $y = -3 \cos 2t - 4 \sin 2t$

2. Consider the undamped oscillator

$$mx'' + kx = 0, \quad x(0) = x_0, \quad x'(0) = v_0.$$

Write the general solution of this initial value problem in the form $x(t) = a \cos \omega t + b \sin \omega t$ (i.e., determine a , b , and ω), and then write it in the form $x(t) = A \cos(\omega t - \phi)$ (i.e., determine A).

3. The function $x(t) = \cos 6t - \cos 7t$ has mean frequency $\bar{\omega} = 13/2$ and half difference $\delta = 1/2$. Thus,

$$\cos 6t - \cos 7t = \cos\left(\frac{13}{2} - \frac{1}{2}\right)t - \cos\left(\frac{13}{2} + \frac{1}{2}\right)t = 2 \sin \frac{1}{2}t \sin \frac{13}{2}t.$$

Use a computer or calculator to plot both $f(t) = \cos 6t - \cos 7t$ and the “envelope” $g(t) = 2 \sin \frac{1}{2}t$ on the same set of axes.

4. Let $\omega_0 = 11$. Use a computer to plot the graph of the function

$$x(t) = \frac{\cos \omega t - \cos \omega_0 t}{\omega_0^2 - \omega^2}$$

for $\omega = 9, 10, 10.5, 10.9, \text{ and } 10.99$ on the time interval $[0, 24]$. Explain how these solutions approach the resonance solution as $\omega \rightarrow \omega_0$.