

Math 2080: Differential Equations

Worksheet 3.6: Variation of parameters

NAME:

Consider the inhomogeneous differential equation $y'' + 9y = \sec 3t$.

(a) Solve the related homogeneous equation, $y_h'' + 9y_h = 0$, by inspection.

(b) If $y_h(t) = C_1y_1(t) + C_2y_2(t)$ is your answer to the previous part, then assume that there is a particular solution to the original ODE of the form $y_p = v_1y_1 + v_2y_2$. Compute y_p' .

(c) The y_p' from the previous part should have four terms; one each with v_1 , v_2 , v_1' , and v_2' . Eliminate the two terms involving v_1' and v_2' by setting their sum equal to zero. This leaves just two terms in y_p' . Now compute y_p'' .

- (d) Plug $y_p = v_1 y_1 + v_2 y_2$ and y_p'' from Part (c) back into the original ODE. Things should cancel, leaving you with an equation involving v_1' and v_2' (no v_1 or v_2).
- (e) Write a system of two first order ODEs involving v_1' and v_2' . One equation comes from Part (c) and the other from Part (d). Write this in matrix form as well.
- (f) Solve for v_1' and v_2' by multiplying your two equations by suitable functions and and or subtracting them to eliminate the other.
- (g) Integrate to find v_1 and v_2 (feel free to use a computer). Write the general solution to the original ODE.