Math 2080: Differential Equations
Worksheet 3.8: Power series solutions

NAME:

In this problem, you will solve the 2nd order linear ODE $y'' + x^2y'' - 4xy' + 6y = 0$ using the power series method.

(a) Assume that the solution has the form $y(x) = \sum_{n=0}^{\infty} a_n x^n$. Plug $y(x)$ back into the ODE.

(b) Shift the coefficients of the term containing x^{n-2}, and then factor out x^n to get $\sum_{n=0}^{\infty}[\cdots]x^n$, and set $[\cdots] = 0$ to get a recurrence relation for a_{n+2} in terms of a_n.

(c) Explictly compute a_n in terms of a_0 and a_1 for $n \leq 5$.

(d) How many distinct *polynomial* solutions are there?

(e) Write the general solution as $y(x) = a_0 y_0(x) + a_1 y_1(x)$, where y_0 and $y_0(x)$ and $y_1(x)$ are the even and odd terms respectively upon setting $a_0 = a_1 = 1$.