## Math 2080: Differential Equations Worksheet 4.8: Stability of phase portraits

## NAME:

In this problem, consider the system of differentiation equations x' = Ax, where  $A = \begin{bmatrix} \alpha & 1 \\ -1 & \alpha \end{bmatrix}$  and  $\alpha$  is a parameter.

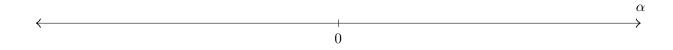
(a) Determine the eigenvalues of  $\mathbf{A}$  in terms of  $\alpha$ .

(b) Find the critical value or values of  $\alpha$  where the qualitative nature of the phase portrait for the system changes.

(c) Draw a phase portrait for a value of  $\alpha$  slight below, and for another value slightly above, each critical value.

(d) Draw a phase portrait when  $\alpha$  is exactly the critical value.

(e) Summarize Parts (b)–(d) by dividing the number line below into regions corresponding to phase portraits of the same types. Clearly label your diagram.



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