

Math 2080: Differential Equations

Worksheet 5.2: Properties & applications of Laplace transforms

NAME:

The following properties of the Laplace transform will be useful in this worksheet:

$$(i) \mathcal{L}\{e^{at}\}(s) = \frac{1}{s-a}$$

$$(iv) \mathcal{L}\{\sin bt\}(s) = \frac{b}{s^2 + b^2}$$

$$(ii) \mathcal{L}\{t^n\}(s) = \frac{n!}{s^{n+1}}$$

$$(v) \mathcal{L}\{e^{ct} f(t)\}(s) = F(s-c)$$

$$(vi) \mathcal{L}\{t^n f(t)\}(s) = (-1)^n \frac{d^n}{ds^n} F(s)$$

$$(iii) \mathcal{L}\{\cos bt\}(s) = \frac{s}{s^2 + b^2}$$

$$(vii) \mathcal{L}\{y''(t)\}(s) = s^2 Y(s) - sy(0) - y'(0)$$

1. Compute the Laplace transform of te^{3t} two ways: using Properties (v) and (vi).

2. Compute the Laplace transform of $e^{2t} \cos 6t$.

3. Compute the inverse Laplace transform of $Y(s) = \frac{3}{2-6s}$. (Factor out -6)

4. Compute the inverse Laplace transform of $Y(s) = \frac{1}{(s-3)(s+1)}$. (Partial fractions)

5. Compute the inverse Laplace transform of $Y(s) = \frac{1}{s^2 + 4s + 13}$. (Complete the square)

6. Compute the inverse Laplace transform of $Y(s) = \frac{s}{s^2 + 4s + 13}$. (Complete the square)

7. In this problem, you will solve the initial value problem: $y'' - y = e^{2t}$, $y(0) = 1$, $y'(0) = 0$.

(a) Take the Laplace transform of the initial value problem and solve for Y .

(b) Use partial fraction decomposition to break up your equation for $Y(s)$.

(c) Take the inverse Laplace transform of each fraction to get the solution to the initial value problem.