

## Math 2080: Differential Equations

### Worksheet 7.2: Different boundary conditions

**NAME:**

1. Let  $u(x, t)$  be the temperature of a bar of length 10 that is fully insulated so that no heat can enter or leave, even at the endpoints. Suppose that initially, the temperature is increasing linearly from  $70^\circ$  at one endpoint to  $80^\circ$  at the other endpoint.

(a) Sketch the initial heat distribution on the bar and express it as a function of  $x$ .

(b) Write down an IVP/BVP to which  $u(x, t)$  is a solution. (Let the constant from the heat equation be  $c^2$ .)

(c) What will the steady-state solution be?

2. Let  $u(x, t)$  be the temperature of a bar of length 10 that is fully insulated at its right endpoint but uninsulated at its left endpoint. Suppose the bar is sitting in a  $70^\circ$  room and that initially, the temperature of the bar increases linearly from  $70^\circ$  at the left endpoint to  $80^\circ$  at the other end. Finally, suppose the interior of the bar is poorly insulated, so heat can escape.

(a) Suppose that heat escapes at a constant rate of  $1^\circ$  per hour. Write an IVP/BVP for  $u(x, t)$  that could model this situation.

(b) A more realistic situation would be for heat to escape not at a constant rate, but at a rate proportional to the *difference* between the temperature of the bar and the ambient temperature of the room. Write an IVP/BVP for  $u(x, t)$  that could model this situation. What is the steady-state solution and why?

3. Consider the following PDE, where  $\gamma$  is a constant, and  $h(x)$  and arbitrary function on  $[0, \pi]$ :

$$u_t = c^2 u_{xx}, \quad u(0, t) = 0, \quad u_x(\pi, t) + \gamma u(\pi, t) = 0, \quad u(x, 0) = h(x),$$

(a) Describe a physical situation that this models. Be sure to describe the impact of the initial condition, both boundary conditions, and the constant  $\gamma$ .

(b) What is the steady-state solution and why? (Use your physical intuition).