## Math 2080: Differential Equations Worksheet 7.3: The transport equation

## NAME:

1. The PDE  $u_{tt} = c^2 u_{xx}$  is called the *wave equation*. Here it is below written in several different ways.

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial x}\right) \left(\frac{\partial}{\partial t} - c\frac{\partial}{\partial x}\right) u = \left(\frac{\partial^2}{\partial t^2} - c\frac{\partial^2}{\partial x^2}\right) u = \frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = u_{tt} - c^2 u_{xx} = 0$$

Let f(x) and g(x) be differentiable functions, and define u(x,t) = f(x+ct) + g(x-ct). Compute  $u_{tt}$  and  $u_{xx}$  and check that u(x,t) solves the wave equation.

2. Consider the following initial value problem for the wave equation:

$$u_{tt} = c^2 u_{xx}, \qquad u(x,0) = f(x), \quad u_t(x,0) = 0.$$

If f(x) is any differentiable function, then define  $u(x,t) = \frac{1}{2}f(x+ct) + \frac{1}{2}f(x-ct)$ .

(a) Let  $f(x) = e^{-x^2/2}$ . Sketch u(x, 0) and u(x, t) for some t > 0.

(b) Compute  $u_t$ ,  $u_{tt}$ , and  $u_{xx}$  and verify that u(x,t) solves the IVP above.