Math 2080: Differential Equations Worksheet 7.6: Laplace's equation

NAME:

In this worksheet, you will solve the following three different instances of Laplace's equation on a square region, where u(x, y) is defined for $0 \le x \le \pi$ and $0 \le y \le \pi$:

- (i) $\nabla^2 u = 0$, $u(x, 0) = u(0, y) = u(\pi, y) = 0$, $u(x, \pi) = \sin x$.
- (ii) $\nabla^2 u = 0$, $u(x, 0) = u(0, y) = u(x, \pi) = 0$, $u(\pi, y) = \sin 2y$.
- (iii) $\nabla^2 u = 0$, u(x, 0) = u(0, y) = 0, $u(x, \pi) = \sin x$, $u(\pi, y) = \sin 2y$.
- (a) Sketch the solutions to each of these three BVPs. *Hint: it is enough to sketch the boundaries, and then use the fact that the solutions are harmonic functions.*

(b) Solve the following BVP from (i) for u(x, y):

$$abla^2 u = 0,$$
 $u(x,0) = u(0,y) = u(\pi,y) = 0,$ $u(x,\pi) = \sin x$

(c) Solve the following BVP from (ii) for u(x, y):

$$\nabla^2 u = 0,$$
 $u(x,0) = u(0,y) = u(x,\pi) = 0,$ $u(\pi,y) = \sin 2y.$

(d) Using superposition, solve following the BVP from (iii) for u(x, y):

$$\nabla^2 u = 0,$$
 $u(x,0) = u(0,y) = 0,$ $u(x,\pi) = \sin x,$ $u(\pi,y) = \sin 2y.$