1. Four of the many common ways of writing the discrete logistic growth equation are

$$\Delta P = rP(1 - P/K), \qquad \Delta P = sP(K - P),$$
$$\Delta P = tP - uP^2, \qquad P_{t+1} = vP_t - wP_t^2\,.$$

Write each of the following in all four of these forms.

(a) $P_{t+1} = P_t + .2P_t(10 - P_t)$

(b) $P_{t+1} = 2.5P_t - .2P_t^2$

2. Consider the difference equation $P_{t+1} = P_t(2 - P_t)$. The graphs of $F(x) = x(2 - x)$, and $x = y$ intersect at $(x, y) = (1, 1)$, which is the local maximum of the parabola.

(a) Draw two cobweb plots, one corresponding to an initial condition $P_0 < 1$ and the other to the initial condition $P_0 > 1$. Recall that these lie on the (P_t, P_{t+1})-plane.

(b) For each of these, plot the points (t, P_t) for a few small integer values of t.

(c) How do the previous parts compare to the case when the point of intersection is to the left of the local maximum? To the right of the local maximum?

3. Consider the following instance of the discrete logistic equation:

$$P_{t+1} = P_t(1 + r(1 - P_t/K))$$

Find the two equilibirum points, P^*. Use the technique of *linearization* to find the stability of these points. That is, plug $P_t \approx P^* + p_t$ and $P_{t+1} \approx P^* + p_{t+1}$ into the difference equation and express the perturbation p_{t+1} in terms of p_t, disregarding the non-linear terms.

4. The discrete logistic and the Ricker population models when written as $P_{t+1} = F(P_t)$ have the property that for small values of P_t, the graph of $F(x)$ lies *above* the line $y = x$ This means that $F(P_t) > P_t$ for small value of P_t. Consider a model for which $F(P_t) < P_t$ for small values of P_t. Explain the affect of this feature on population dynamics. Why might this be a biologically important feature? (The resulting behavior is sometimes known as an *Allee effect.*)

5. Construct a simple model showing an Allee effect as follows.

(a) Explain why for some $0 < L < K$, the per-capita growth should be

$$\frac{\Delta P}{P} < 0, \quad \text{when } 0 < P < L \ \text{ or } P > K\,,$$
$$\frac{\Delta P}{P} > 0, \quad \text{when } L < P < K\,.$$

Sketch a possible graph of $\Delta P/P$ vs. P.

(b) Explain why $\Delta P/P = P(K - P)(P - L)$ has the qualitative features desired.

(c) Investigate the resulting model using the MATLAB programs `onepop` and `cobweb` (available on the Math 4500 webpage) for some choices of K and L. Print out, or sketch the results of a few sample trials. Is the behavior as expected?

(d) What features of this modeling equation are unrealistic? How might the model be improved?