Modeling biochemical reactions

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- E_0 is constant.
- Enzyme-substrate complex reaches equilibrium much earlier than the product does, so $\dfrac{d[ES]}{dt} \approx 0$.

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The Michaelis–Menten equation is one of the best-known models of enzyme kinetics.

$$\frac{d[P]}{dt} = \underbrace{\frac{V_{\max}[S]}{K_m + [S]}}_{f([S])}, \qquad \text{where } V_{\max} = k_3 E_0, \quad \text{and } K_m = \frac{k_2 + k_3}{k_1}$$

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$$\frac{d[ES_n]}{dt} = 0 \qquad \Longrightarrow \qquad [E] = \frac{(k_2 + k_3)[ES_n]}{k_1[S]^n}.$$

Multi-molecule binding

Consider a reaction where n molecules of a substrate S react with an enzyme E:

$$E + nS \underset{k_2}{\overset{k_1}{\rightleftharpoons}} ES_n \xrightarrow{k_3} E + P$$

The enzyme-substrate complex here is ES_n. By mass-action kinetics,

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Multi-molecule binding

Multi-molecule binding

Hill equation

Given the chemical reaction

$$E + nS \underset{k_1}{\overset{k_2}{\rightleftharpoons}} ES_n \xrightarrow{k_3} E + P$$

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- $n = 1$ is just the Michaelis–Menden equation.

Hill equations

Hill equations

The following shows several "Hill functions" $y = \dfrac{t^n}{1+t^n}$, for various values of n.