Cellular automata and agent-based models

Matthew Macauley

Department of Mathematical Sciences
Clemson University
http://www.math.clemson.edu/~macaule/

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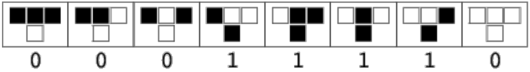

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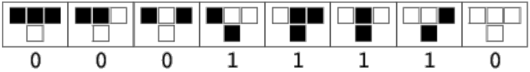
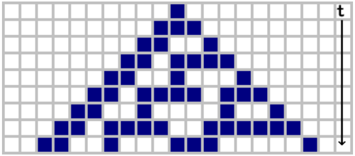


The following shows the evolution of the dynamics over $t = 0, 1, \ldots, 8$, starting with a single "ON" cell:



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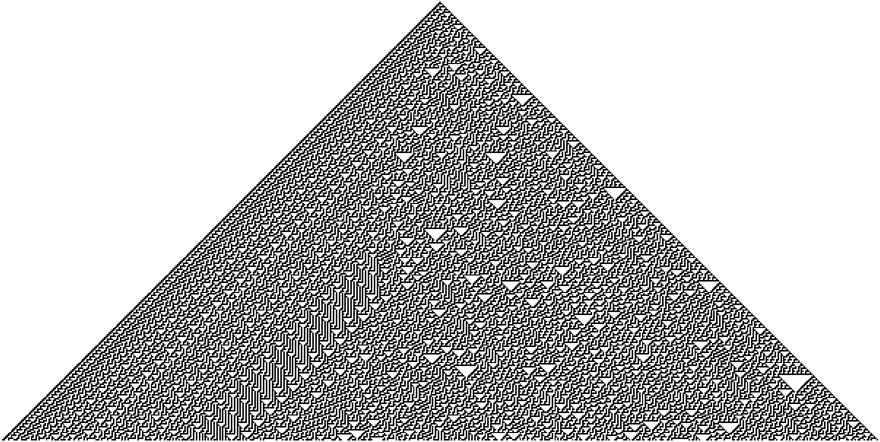

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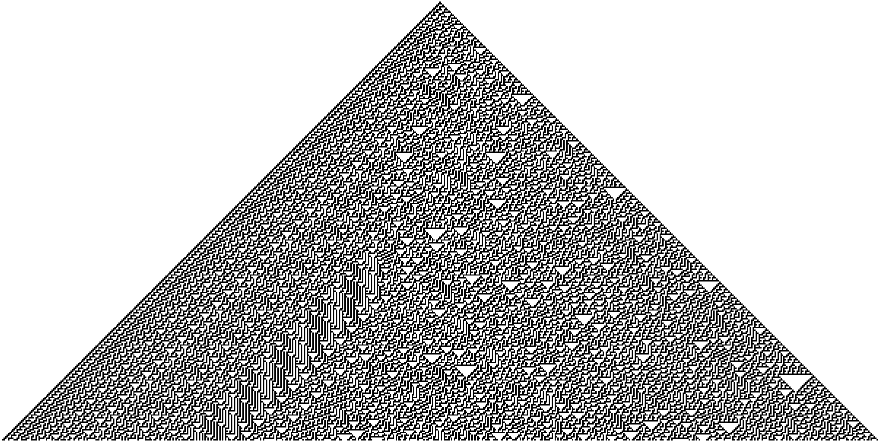

A common theme with CA are that complex dynamics can emerge from simple, local interactions.

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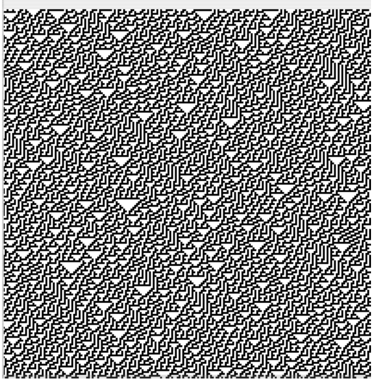

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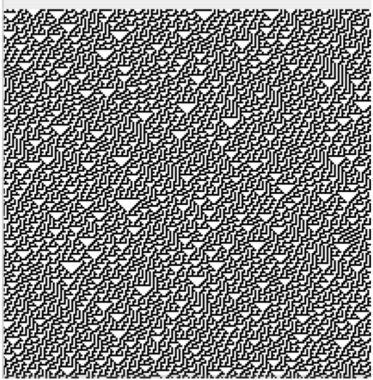

Many believe that CAs are key to understanding how simple rules can produce complex structures and behavior.

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There are many variations, leading to an endless potential of research projects.

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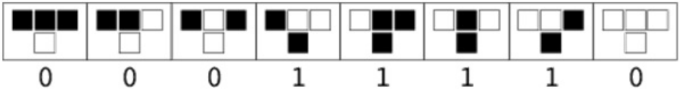
For example, what ECA rule is this?



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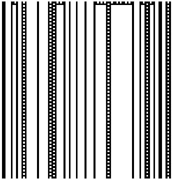

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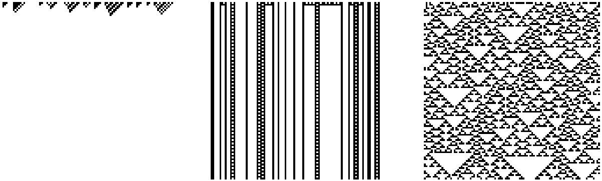

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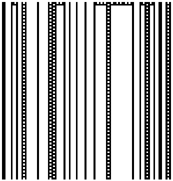

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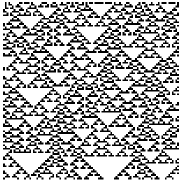

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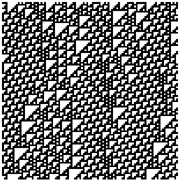

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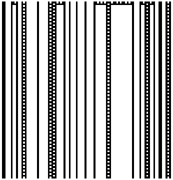

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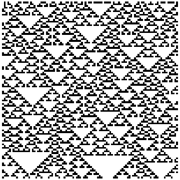

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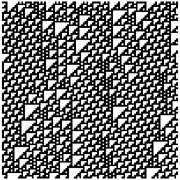

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Problem

Class membership of a given rule is computationally *undecidable*!

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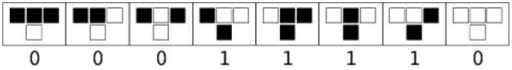

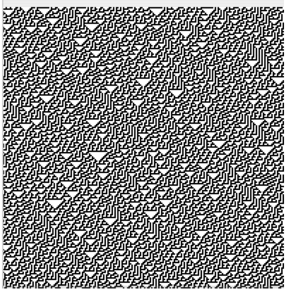

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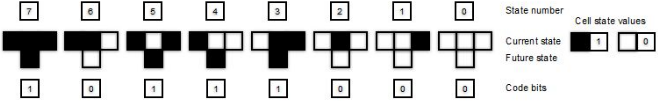

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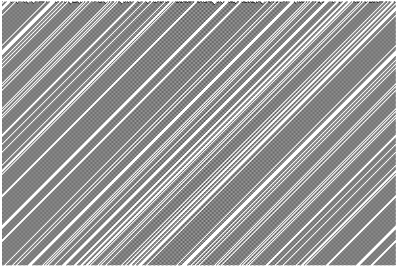

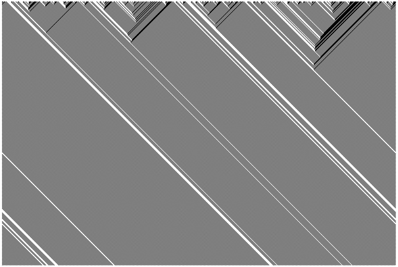

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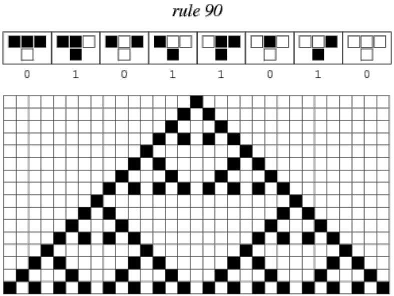

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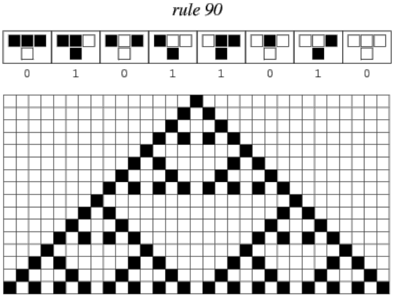

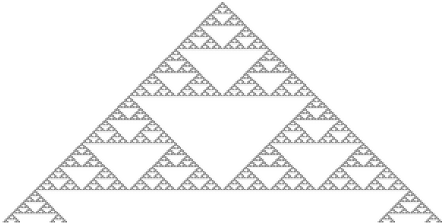

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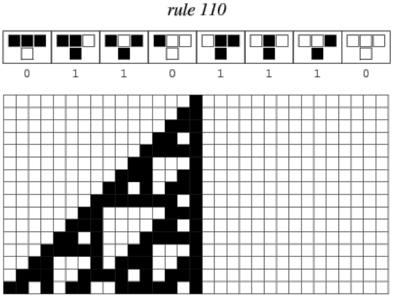

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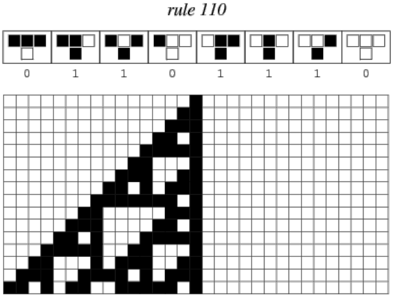

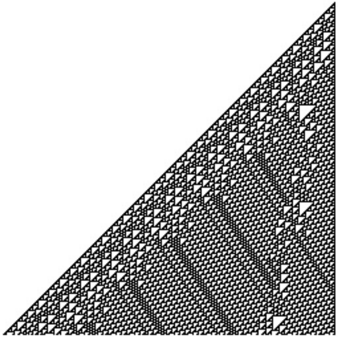

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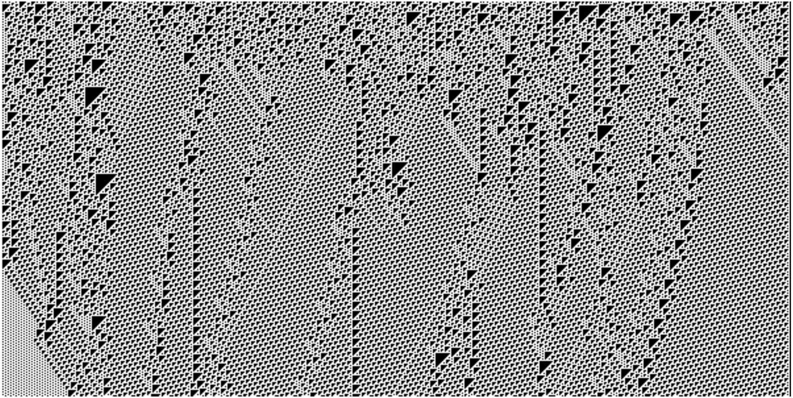
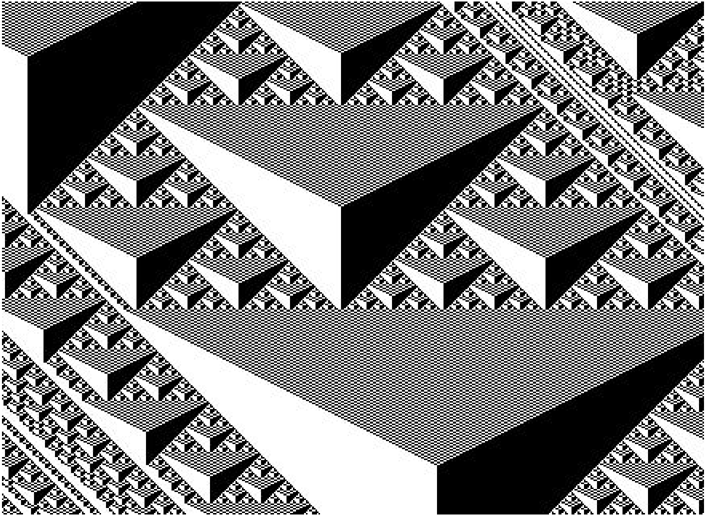


Generalizations of elementary cellular automata

There are $2^{2^5} = 4294967296$ one-dimensional "2-neighbor rules". Here is one of them:



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- Exploring the universe would be easier if machines could be invented that could build themselves. Imagine sending a probe to Mars that could build a copy of itself.

Other agent-based models

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Cellular automata promote the idea that complex dynamics can arise from simple local rules.

Agent based model of flocking

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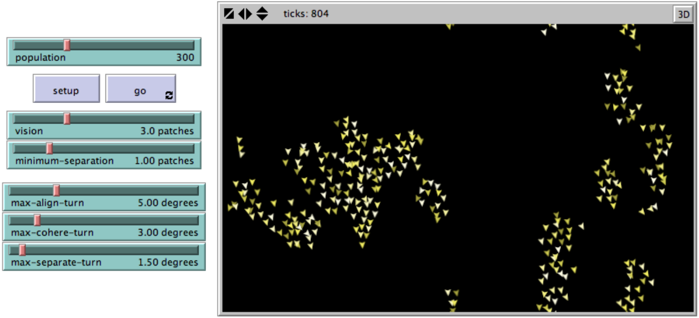
NetLogo is a wonderful free open-source program that has many built-in agent-based models such as this one.



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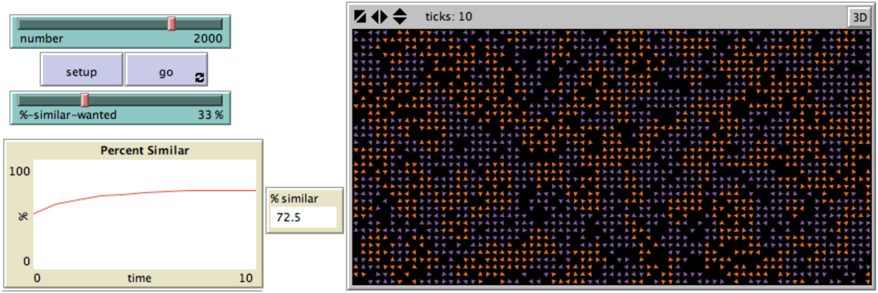
Here is a result of a NetLogo simulation showing where the tiger end up living.



Other agent-based models in NetLogo

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NetLogo has hundreds of built-in agent-based models, such as:

- firefly synchronization
- voting communities
- percolation (oil spill through soil)
- traffic grids
- forest fires
- moths and light
- enzyme kinetics
- tumor growth
- virus spreading through a network
- shepherds herding sheep
- predator-prey
- ⋮