Math 2080: Differential Equations
Worksheet 1.1: What is a differential equation?

NAME:

1. Consider an investment that grows at a constant interest rate of 5%, compounded continuously. Suppose that initially, the investment is worth \$1000. If $P(t)$ is the value of the investment, write out an *initial value problem* (IVP) – a differential equation with an initial condition – that $P(t)$ satisfies and sketch the solutions.

2. Consider an investment that grows at a constant interest rate of 5%, compounded continuously. Suppose that initially, the investment is growing at a *rate* of \$100/year. If $P(t)$ is the value of the investment, write out an IVP that $P(t)$ satisfies and sketch the solutions.

3. The mass $m(t)$ of a radioactive substance decays at a rate proportional to the amount remaining. Suppose there are 100 grams initially. Write out an IVP that $m(t)$ satisfies and sketch the solutions.

4. The temperature $T(t)$ of a cup of coffee cools at a rate proportional to the difference in its temperature with the ambient room temperature (say, 70°). Suppose that the coffee is initally 190°.

   (a) Write out an initial value problem that $T(t)$ satisfies and sketch the solutions.

   (b) Let $y(t) = T(t) - 70$, which represents the number of degrees that the coffee is above or below the room temperature. Substituting $T = y + 70$ back into your IVP from the previous part to get a much simplier IVP in terms of the function $y(t)$ and sketch the solutions.