Math 2080: Differential Equations
Worksheet 4.4: Solving a 2×2 system of ODEs

NAME:

Consider the system of differential equations: $\begin{cases} x_1' = -2x_1 + x_2 - 12, & x_1(0) = 5 \\ x_2' = 4x_1 + x_2 - 18, & x_2(0) = 20 \end{cases}$

1. Write this in matrix form, $\boldsymbol{x}' = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}$, and find the steady-state solution, $\boldsymbol{x}_{ss}$.

2. Make a change of variables to transform the system into a homogeneous system.

3. Solve the homogeneous system. The eigenvalues of $\boldsymbol{A}$ are $\lambda_1 = -3$, $\lambda_2 = 2$, with eigenvectors $\boldsymbol{v}_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and $\boldsymbol{v}_2 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$.

4. Solve the solution to the original system of differential equations, by writing $\boldsymbol{x} = \boldsymbol{x}_h + \boldsymbol{x}_{ss}$.

5. Sketch the phases portrait of the homogeneous system, and the inhomogeneous system. Also sketch the particular solutions satisfying the given initial conditions.