

## Math 2080: Differential Equations

### Worksheet 4.6: Phase portraits with complex eigenvalues

**NAME:**

Consider the system of differential equations:  $\begin{cases} x_1' = 3x_1 + 2x_2, & x_1(0) = 0 \\ x_2' = -5x_1 + x_2, & x_2(0) = 1 \end{cases}$

(a) Write this in matrix form,  $\mathbf{x}' = \mathbf{A}\mathbf{x}$ ,  $\mathbf{x}(0) = \mathbf{x}_0$ .

(b) Given that the eigenvalues of  $\mathbf{A}$  are  $\lambda_1 = 2 + 3i$  and  $\lambda_2 = 2 - 3i$ , with associated eigenvectors  $\mathbf{v}_1 = \begin{bmatrix} -1 - 3i \\ 5 \end{bmatrix}$  and  $\mathbf{v}_2 = \begin{bmatrix} -1 + 3i \\ 5 \end{bmatrix}$ , write the general solution to  $\mathbf{x}' = \mathbf{A}\mathbf{x}$ .

(c) Write the general solution as a linear combination of *real-valued* functions:  $\mathbf{x}(t) = C_1\mathbf{u}(t) + C_2\mathbf{w}(t)$ .

- (d) Find the particular solution satisfying the initial condition.
- (e) The phase portrait will consist of spiraling ellipses. To determine whether the spirals are clockwise or counterclockwise, compute the  $\mathbf{x}'(0) = \begin{bmatrix} x_1'(0) \\ x_2'(0) \end{bmatrix}$  and see which direction it points.
- (f) Sketch the phase portrait of the system, and sketch the particular solution satisfying the initial condition. Feel free to use a computer to plot the approximate tilt and shape of the ellipse.