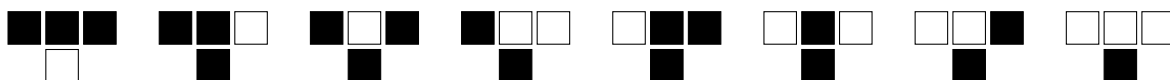


Read:

- Chapter 4.1: Global dynamics emerging from local interactions: agent-based modeling for the life sciences. By D. Gammack, Elsa. Schaefer, and H. Gaff, pages 105–113.
  - Chapter 5.1–5.4: Agent-based models and optimal control in biology: a discrete approach. By R. Laubenbacher, F. Hinkelmann, and M. Oremland, pages 143–152.
1. Download *Cellular Automaton Explorer* from David Bahr’s homepage. Under the “Rule” tab, select “Wolfram 1-d rules”. Under the “Lattice and Properties” tab, configure the settings to have 150 rows and columns. Now, do the following for ECA Rules 54, 73, 137, and one more “interesting” rule of your choice.

- (a) Explore the dynamics of the elementary cellular automaton (ECA) rule using both a single black square as a starting state, and using a random state. Print a screenshot of your findings.
- (b) Which of the four Wolfram classes do you think this rule belongs to, and why?

2. Consider the ECA rule defined by the following *truth table*:



Here, white = 0, black = 1, and each 4-box diagram represents a three-bit input vector  $(x_{i-1}(t), x_i(t), x_{i+1}(t))$  and the corresponding output  $x_i(t+1) = f(x_{i-1}(t), x_i(t), x_{i+1}(t))$ .

- (a) Find the decimal form of this rule. It should be a number between 0 and 255.
  - (b) Write the local function  $f(x_{i-1}, x_i, x_{i+1})$  as a polynomial over  $\mathbb{Z}_2 = \{0, 1\}$ .
  - (c) Write the local function  $f(x_{i-1}, x_i, x_{i+1})$  as a Boolean logical expression (that is, using  $\vee$ ,  $\wedge$ , and  $\neg$ ).
3. Consider the *parity function*  $f(x_{i-1}, x_i, x_{i+1}) = x_{i-1} + x_i + x_{i+1}$ , where addition is assumed to be done modulo 2.
    - (a) Write out the truth table of this function.
    - (b) Find this rule’s decimal representation.
    - (c) Write this rule using Boolean logic.
  4. Find a 1-dimension ECA rule that always translates its initial state one position to the right. Express it four different ways: by its truth table, in decimal form, using Boolean logic, and as a polynomial over  $\mathbb{Z}_2$ .
  5. Download NetLogo from <https://ccl.northwestern.edu/netlogo/>. Find an agent-based model that we did not try out in class. Adjust one of the parameters until you find a “threshold” between two distinct types of behavior. Describe your findings in a paragraphs, including the approximate value of the threshold, and print out a screen shot (in color, if possible).