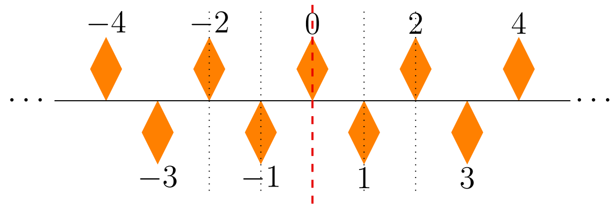
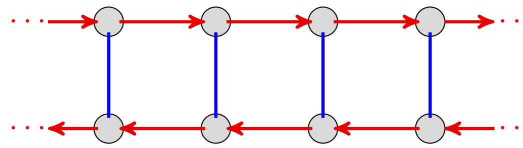
1. Consider the frieze pattern below at left, with the diamonds labeled for reference.





Let g be a glide-reflection to the right, and h a horizontal flip about the dashed red line. These actions generate the symmetry group of this frieze, and its Cayley diagram is shown on the right. For each of the four dotted lines shown in the frieze, consider the reflection across that line and the resulting re-numbering of the nine diamonds pictured. Then do the following:

   (a) Redraw the frieze above and re-number the diamonds accordingly.
   (b) Express this reflection in terms of g and h.

   Write down two presentations for this frieze group: one generated by g and h, and another by h and the reflection across the diamond numbered 1. How is this group related to D_n?

2. Let G be a finite group.
   (a) Show that for any $n > 2$, the number of elements in G of order n is even.
   (b) If $|G|$ is even, show that G has an element of order 2.

3. Let $K \leq H \leq G$ be groups. Show that $[G : K] = [G : H][H : K]$.

4. Let H be a subgroup of G.
   (a) Show that gHg^{-1} is a subgroup, and that $H \cong gHg^{-1}$.
   (b) Show that in any group, $|xy| = |yx|$.
   (c) Show that if G has a unique element x of order 2, then $x \in Z(G)$.

5. Let $|G| = \infty$, and $[G : H] < \infty$. Show that H intersects every infinite subgroup of G nontrivially.

6. If $A, B \leq G$ and $y \in G$, define the (A, B)-*double coset* $AyB = \{ayb \mid a \in A,\ b \in B\}$.
   (a) Show that G is the disjoint union of its (A, B)-double cosets.
   (b) Show that if A and B are finite, then $|AyB| = [A^y : A^y \cap B] \cdot |B|$, where $A^y = y^{-1}Ay$.

7. Prove that if A and B are normal subgroups of G and $AB = G$ then
$$G/(A \cap B) \cong (G/A) \times (G/B).$$