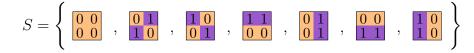
1. Let S be the following set of 7 "binary squares":



For the following actions, draw an action diagram and find the stabilizer of each element:

- (a) The action of $G = V_4 = \langle v, h \rangle$ on S, where $\phi(v)$ reflects each square vertically, and $\phi(h)$ reflects each square horizontally.
- (b) The action of $G = C_4 = \langle r \rangle$ on S, where $\phi(r)$ rotates each square 90° clockwise.
- 2. Let $G = S_4$ act on itself by conjugation via the homomorphism

 $\phi \colon G \longrightarrow \operatorname{Perm}(S), \qquad \phi(g) = \operatorname{the permutation that sends each } x \mapsto g^{-1}xg.$

- (a) How many orbits are there? Describe them as specifically as you can.
- (b) Find the orbit and the stabilizer of the following elements:
- i. id ii. (1 2) iii. (1 2 3) iv. (1 2 3 4) v. (1 2) (3 4).
- 3. Prove that if $H \leq S_n$ contains an odd permutation, then exactly half of its permutations are odd.
- 4. Suppose $H, K \leq G$ both have finite index.
 - (a) Show that there is some $N \leq G$ with $N \leq H$ such that $[G:N] < \infty$.
 - (b) Show that $[G: H \cap K] \leq [G: H][G: K]$.
- 5. Let G be a finite group acting on a set S.
 - (a) Prove that if G has no subgroup of index 2, then any subgroup of index 3 is normal.
 - (b) Prove that if [G:H] = p, the smallest prime dividing |G|, then $H \leq G$.
- 6. Given a left action, show that $\operatorname{Stab}_G(xs) = x \operatorname{Stab}_G(s) x^{-1}$ for all $x \in G, s \in S$.
- 7. Permutation groups G_1 and G_2 acting on sets S_1 and S_2 are called *permutation isomorphic* if there exists an isomorphism $\theta: G_1 \to G_2$ and a bijection $\phi: S_1 \to S_2$ such that $(\theta x)(\phi s) = \phi(xs)$ for all $x \in G_1$ and $s \in S_1$. In other words, the following diagram commutes:

$$\begin{array}{c|c} S_1 & \xrightarrow{x} & S_1 \\ \phi & & & \downarrow \phi \\ S_2 & \xrightarrow{\theta x} & S_2 \end{array}$$

Show that the following two actions of G on itself are permutation isomorphic:

- (i) the action of $x \in G$ is left multiplication by x;
- (ii) the action of $x \in G$ is right multiplication by x^{-1} .