1. For each of the following statements, prove or give a counterexample.
   (a) Let $f\colon G \to H$ be an epimorphism. For any two homomorphsims $g_1, g_2\colon H \to K$, the equality $g_1 \circ f = g_2 \circ f$ implies that $g_1 = g_2$.
   (b) Let $f\colon G \to H$ be a monomorphism. For any two homomorphsims $g_1, g_2\colon H \to K$, the equality $g_1 \circ f = g_2 \circ f$ implies that $g_1 = g_2$.
   (c) Let $g\colon H \to K$ be an epimorphism. For any two homomorphsims $f_1, f_2\colon G \to H$, the equality $g \circ f_1 = g \circ f_2$ implies that $f_1 = f_2$.
   (d) Let $g\colon H \to K$ be a monomorphism. For any two homomorphsims $f_1, f_2\colon G \to H$, the equality $g \circ f_1 = g \circ f_2$ implies that $f_1 = f_2$.

2. If (U, ε) is a universal pair for a group G and $h \in \mathrm{Aut}(U)$, show that $(U, h\varepsilon)$ is also universal for G. Conversely, if (U, ε_1) is universal for G, show that $\varepsilon_1 = h\varepsilon$ for some $h \in \mathrm{Aut}(U)$.

3. For each of the following groups G:

   (i) D_3, (ii) D_4, (iii) Q_8, (iv) A_4, (v) S_4,

   carry out the steps below. Feel free to use Google for Part (a).
   (a) Draw the subgroup lattice of G and circle each normal subgroup.
   (b) Determine the commutator subgroup G' and double-circle this on the lattice.
   (c) Sketch the subgroup lattice of G/G' and find its isomorphism type.
   (d) Find the derived series for G.

4. Prove the lemma from class:
   (a) If $G' \leq H \leq G$ show that $H \lhd G$.
   (b) Show that if $K \lhd G$, then $K' \lhd G$.
   (c) Suppose $f\colon G \to H$ is an epimorphism, with $\ker f = K$. Show that H is abelian if and only if $G' \leq K$.

5. Prove that G is solvable in the following cases, where p and q are distinct primes:
   (a) $|G| = p^n$,
   (b) $|G| = p^2 q$.

6. Let G be a finite group and $N \trianglelefteq G$.
   (a) If the order of $xN \in G/N$ is a power of p, show that there exists $y \in G$ such that $|y|$ is a power of p and $yN = xN$.
   (b) If G/N is abelian and P is a Sylow p-subgroup of G, prove that PN/N is the unique Sylow p-subgroup of G/N.
   (c) Show by example how Part (b) can fail if G/N is non-abelian.