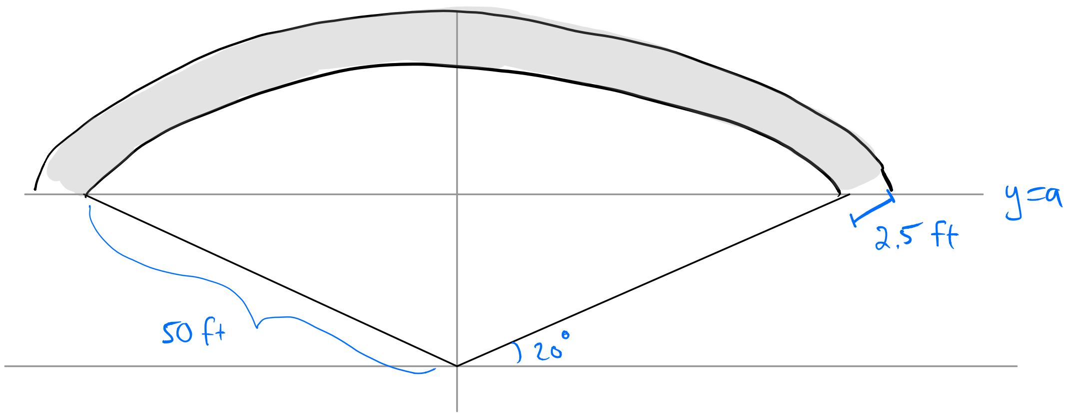
(1.) <u>Hagia Sophia</u> : See PPT slides for history and overview

Our goal is to calculate the weight of the dome.

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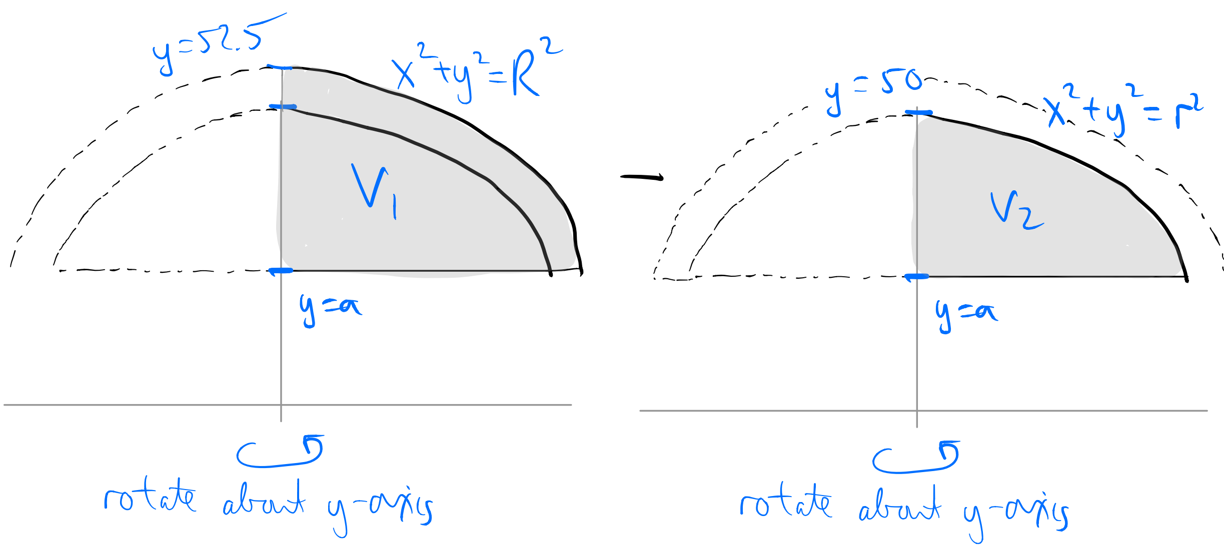
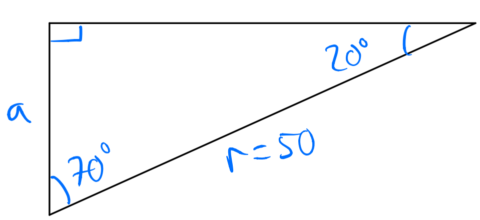



<u>Find a</u>:

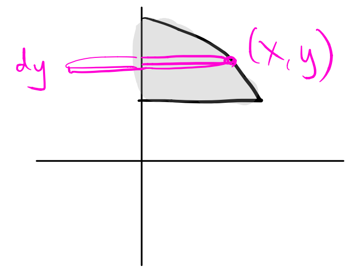



$$\cos 70^\circ = \frac{a}{50}$$

$$\Rightarrow a = 50 \cos 70 \approx 17.1$$

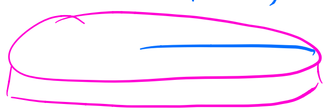
So $R=52.5$, $r=50$, $a=17.1$

$\boxed{2}$




## Find $V_1$:

radius = x-value = $\sqrt{R^2-y^2}$



$$V_1 = \int_a^R \pi\,(\text{radius})^2\,dy = \int_a^R \pi\sqrt{R^2-y^2}\,dy$$

$$= \int_a^R \pi\,(R^2-y^2)\,dy = \pi\left(R^2 y - \frac{y^3}{3}\right)\Bigg|_{a=17.1}^{R=52.5}$$

$$= \pi\left(R^3 - \frac{R^3}{3}\right) - \pi\left(R^2 a - \frac{a^3}{3}\right)$$

$$= \pi\left[\frac{2}{3}R^3 - R^2 a + \frac{1}{3}a^3\right] \simeq 160233 \text{ ft}^3$$

$$V_2 = \int_a^r \pi\,(\text{radius})^2\,dy = \int_a^r \pi\sqrt{r^2-y^2}\,dy$$

$\leftarrow$ same as $V_1$ but replace $R$ with $r$

$$\ldots = \pi\left[\frac{2}{3}r^3 - r^2 a + \frac{1}{3}a^3\right] \approx 132733 \text{ ft}^3$$

So $V_1 - V_2 = 27500 \text{ ft}^3$

The concrete in the dome weighs $\simeq 110$ lb/ft$^3$

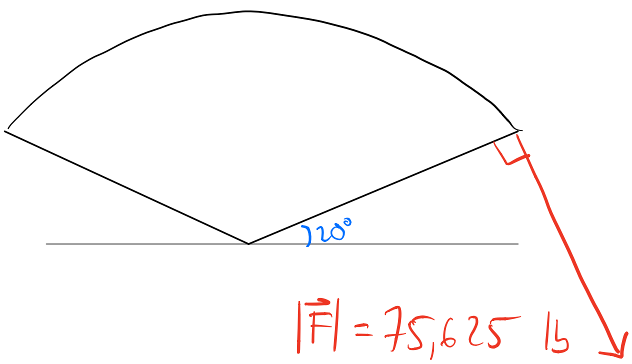
$\Rightarrow$ total weight is $(27500 \text{ ft}^3)(110 \text{ lb/ft}^3)$

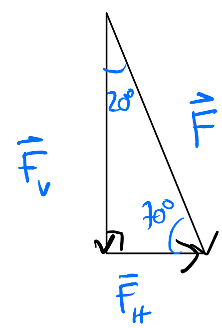
$$= \boxed{3{,}025{,}000 \text{ lb}}.$$

There are 40 supporting ribs $\Rightarrow \frac{3,025,000}{40} = 75,625$ lbs/rib.

Let's calculate the outward force at each buttress

Wed 11/20



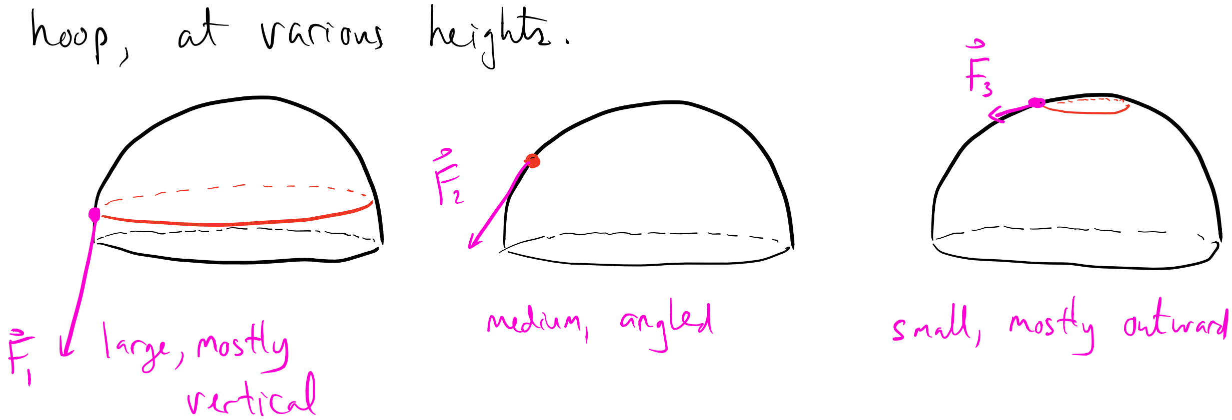

Basic trig: $|\vec{F}_v| = |\vec{F}| \cdot \cos 20 \approx 71,064$ lb

$|\vec{F}_h| = |\vec{F}| \cos 70 \approx 25,865$ lb

Def: • Vertical cross-sections of a dome are called meridians

• Horizontal cross-sections of a dome are called hoops

Compare the force (magnitude & direction) of the dome on a hoop, at various heights.



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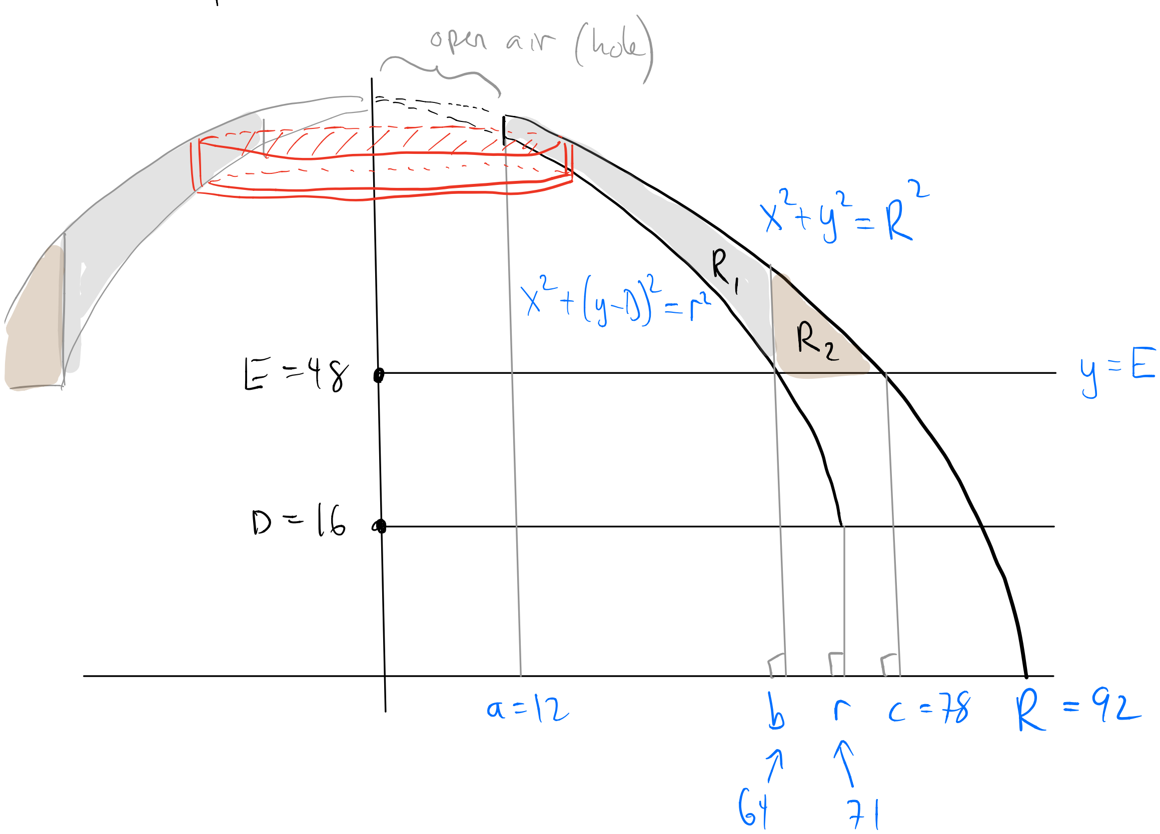
The hoop stress is the outward force.

★ First principle of structural architecture: Unless the shell can resist the "hoop stress," it will expand along the hoops & cracks will develop along meridians.

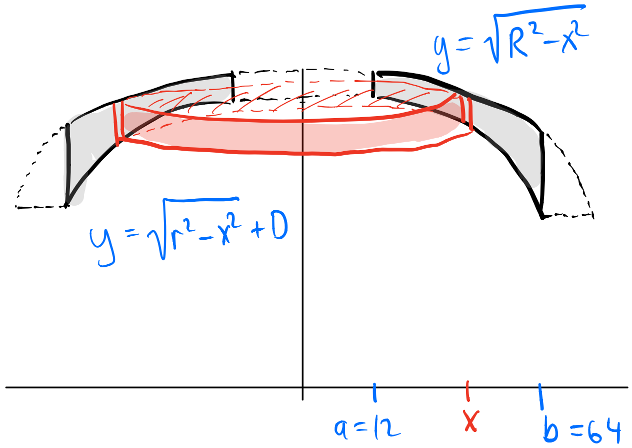
② Roman Pantheon: See PPT slides for history & overview.

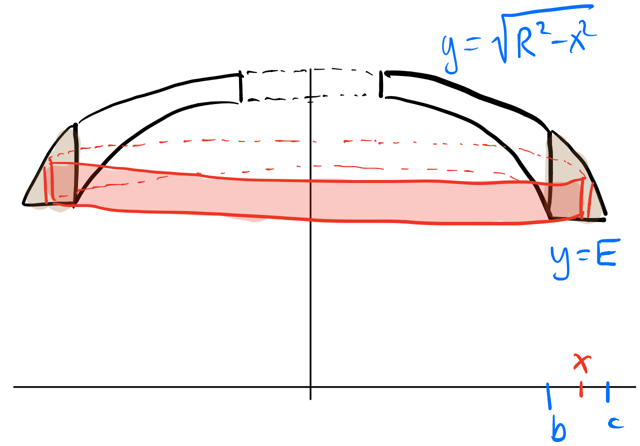
Two key differences b/w Pantheon & Hagia Sophia:

1. The inner & outer shells are spheres with different centers
2. The brick & mortar was mixed with pumice at the top to reduce hoop stress.

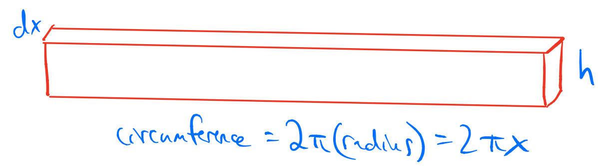


We'll use the <u>shell method</u>




<u>Recall</u>: Vol of a shell




$$\text{Vol } 1 = \int_a^b 2\pi x \left[\sqrt{R^2-x^2} - \left(\sqrt{r^2-x^2}+D\right)\right] dx$$

Let $u = R^2 - x^2$

$\Rightarrow du = -2x\, dx$

$$= \int_a^b 2\pi x\sqrt{R^2-x^2}\, dx - \int_a^b 2\pi x\sqrt{r^2-x^2}\, dx - \int_a^b 2\pi x D\, dx$$

$$= \int_{x=a}^{x=b} -\pi\sqrt{u}\, du + \int_{x=a}^{x=b} \pi\sqrt{u}\, du - \int_a^b (2\pi D)\, x\, dx$$

$$= -\frac{2}{3}\pi u^{3/2}\Big|_{x=a}^{x=b} + \frac{2}{3}\pi u^{3/2}\Big|_{x=a}^{x=b} - \pi D x^2\Big|_a^b$$

$$= -\frac{2}{3}\pi\left(R^2-x^2\right)^{3/2}\Big|_a^b + \frac{2}{3}\pi\left(r^2-x^2\right)^{3/2}\Big|_a^b - \pi D\left(b^2-a^2\right)$$

$$= -656{,}875 + 984{,}818 - 198{,}694 = \boxed{129{,}299 \text{ ft}^3}$$